

DYNAMIC PARAMETER IDENTIFICATION OF PARALLEL KINEMATIC
MACHINES USING THE UNSCENTED KALMAN FILTER

By

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This work is dedicated to my lovely wife, Eunyoung Cho, and my parents, Chongchul Oh and Ohsoon Kwon, whose love and support made all my accomplishments possible and still motivate me.

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TABLE OF CONTENTS

	<u>Page</u>
ACKNOWLEDGMENTS	iv
LIST OF TABLES	viii
LIST OF FIGURES	x
NOMENCLATURE.....	xvii
 CHAPTERS	
1 INTRODUCTION	1
1.1 History of Parallel Kinematic Machines.....	1
1.2 Types of PKM and Applications	2
1.3 PKM Kinematics	4
1.4 PKM Dynamics and Controls.....	4
1.5 Prior Work within SAMM Lab.....	5
1.6 System Identification Methods	6
1.7 Problem Statement.....	7
 2 THEORETICAL BACKGROUND.....	 11
2.1 System Description and Model.....	12
2.1.1 System Description.....	12
2.1.1.1 UF-PKM.....	12
2.1.1.2 SNL-PKM	13
2.1.1.3 FORD-PKM	13
2.1.2 Lumped Mass Parameter Model.....	14
2.1.3 Coordinate System Assignment	16
2.1.3.1 Global and Moving Platform Coordinate Systems.....	16
2.1.3.2 Strut Coordinate System.....	17
2.2 Equations of Motion	18
2.2.1 Generalized Coordinates	19
2.2.2 Kinematics	20
2.2.3 Kane's Equation	26
2.2.4 Equations of Motion (Force-Based Model).....	29
2.2.5 The Energy Method.....	31
2.3 Parameter Identification Algorithms	33

2.3.1 Least Squares Method (LS)	34
2.3.2 Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF)	35
2.3.2.1 Extended Kalman Filter Parameter Estimation	36
2.3.2.2 UKF Parameter Estimation	37
2.3.2.3 UKF Parameter Estimation Procedure	38
2.4 Implementation Issues	40
2.4.1 The Characterization of PKM Dynamics in Terms of the System Identification	41
2.4.1.1 System Model	41
2.4.1.2 Trajectory Selection	41
2.4.1.3 Metrics of Identifiable Terms and Effort Expended	42
2.4.1.4 Dynamic Force Contributions	44
2.4.1.5 Identifiable metrics parameters	45
2.4.1.6 Summary	47
2.4.2 The Parameter Sensitivity in Sensor Location	48
2.4.2.1 Definition of Sensitivity	48
2.4.2.2 Trajectory Selection and Conditions	50
2.4.2.3 Sensitivity Results	50
3 IMPLEMENTATION OF PARAMETER IDENTIFICATION METHODS	68
3.1 Pendulum Setup and Equations of Motion	68
3.2 The Least Squares Method	70
3.3 The Unscented Kalman Filter Setting	73
3.4 Results	74
4 IMPLEMENTATION OF UKF FOR PARAMETER IDENTIFICATION OF	84
4.1 PKM Simulations (Force-Based)	85
4.2 PKM Simulations (Energy-Based Model)	87
4.3 Simulation Setup	88
4.4 Simulation Results (Force-Based, Without Friction)	89
4.5 Simulation Results (Force-Based, With Friction)	90
4.6 Simulation Results (Energy-Based, Without Friction)	91
4.7 Simulation Results (Force-Based, Without Friction, Original/Random/Spread)	92
4.8 Comparison Between Force- and Energy-Based with Noisy Data	94
4.9 Summary of Results	95
5 PKM EXPERIMENT RESULTS	127
5.1 Experimental Setup	128
5.2 Challenges to Experimental Validation and Reconciliation	129
5.3 Experiment Results (Without Gearbox)	131
5.3.1 Friction Extraction and Estimation Stages	132
5.3.2 Experimental Results with Friction Estimates (Stages 1 and 2)	134
5.3.3 Experimental Results with Different Initial Values (Stage 3)	136
5.4 Summary of Results	137

6	CONCLUSION.....	154
APPENDIX		
A	THE Z-X-Z EULER ANGLE DEFINITION AND [U] MATRIX FORMATION .	158
B	SUPPLEMENTAL DERIVATION DETAILS OF THE EQUATIONS OF	163
C	FIGURES FOR THE COMPARISON BETWEEN FORCE-BASED MODEL	171
LIST OF REFERENCES.....		177
BIOGRAPHICAL SKETCH.....		181

LIST OF TABLES

<u>Table</u>	<u>page</u>
2.1 Grouping of Terms by contributions to $\mathbf{JF}_a$	52
2.2 Motion Trajectory Cases for Section 2.2	53
2.3 Motion Trajectory Cases for Section 2.4.....	53
2.4 Table of Results for Section 2.4.....	54
3.1 Comparison Between UKF and LS Without Noise	77
3.2 Comparison Between UKF and LS With Noise	77
4.1 Calculated Friction Value from Individual Strut Testing for ‘Without Gearbox’	97
4.2 Initial Conditions	97
4.3 Trajectories in Simulation (Four cases).....	97
4.4 Mass Value (Units – kilogram)– Force-Based, Without Friction.....	98
4.5 Inertia Value (all times 1×10^{-6} , Units – Nm^2) – Force-Based, Without Friction	98
4.6 Mass Value (Units – kilogram)– Force-Based, With Friction.....	99
4.7 Inertia Value (all times 1×10^{-6} , Units – Nm^2) – Force-Based, With Friction	99
4.8 Mass Value (Units – kilogram)– Energy-Based, Without Friction	100
4.9 Inertia Value (all times 1×10^{-6} , Units – Nm^2) – Energy-Based, Without Friction.....	100
4.10 Mass Value (Units – kilogram)– Force-Based, Without Friction Randomized	101
4.11 Inertia Value (all times 1×10^{-6} , Units – Nm^2) – Force-Based, Without Friction ...	101
4.12 Mass Value (Units – kilogram)– Force-Based, Without Friction Spread case	102
4.13 Inertia Value (all times 1×10^{-6} , Units – Nm^2)– Force-Based, Without Friction	102
4.14 Mass Values with Noisy Data (Units – kilogram)– Force-Based, Without	103

4.15 Inertia Values with Noisy Data (all times 1×10^{-6} , Units – Nm^2) – Force-Based	103
4.16 Mass Values with Noisy Data (Units – kilogram)– Energy-Based,Without	104
4.17 Inertia Values with Noisy Data (all times 1×10^{-6} , Units – Nm^2) – Energy-Base.....	104
5.1 Friction Estimation with CAD Based Inertia Assumed True Values (Stage 1-A)	140
5.2 Estimated Inertia Values Using Estimated Friction from Stage 1-A Data	141
5.3 Estimated Friction Value Using Previously Estimated Friction Value from	142
5.4 Estimated Inertial Value Using Previously Estimated Friction Value from	143
5.5 Estimated Friction Value Using Previously Estimated Parameters as Initial Value	144
5.6 Estimated Inertial Value Using Previously Estimated Value	145

LIST OF FIGURES

<u>Figure</u>	<u>page</u>
1.1 Stewart Platform [1].....	9
1.2 Spherical Parallel Manipulator [3].....	9
1.3 Novel 6-DOF Parallel Machine [5].....	9
1.4 Turin Parallel Manipulator (TuPaMan) [6].....	9
1.5 Generalized Stewart Platform Schematic [36].....	10
2.1 Schematic of a Stewart-Gough PKM.....	55
2.2 Physical Diagram of Strut i.....	55
2.3 Lumped Mass Parameter Diagram for Strut i.....	56
2.4 PKM Configuration Variants.....	57
2.5 Coordinate System Assignment.....	58
2.6 Naming Conventions	58
2.7 Strut Coordinate System	59
2.8 Flowchart for the UKF.....	60
2.9 Block Diagram of Unscented Transformation [45]	61
2.10 Results Defined by Equations 2.76 through 2.81 for Trajectories of Table 2.2	62
2.11 Results Defined by Equations 2.76 through 2.81 for Trajectories of Table 2.2	63
2.12 Results Defined by Equations 2.76 through 2.81 for Trajectories of Table 2.2	64
2.13 Results Defined by Equations 2.76 through 2.81 for Trajectories of Table 2.2	65
2.14 Results Defined by Equations 2.82 through 2.88 for Trajectories of Table 2.2	66
2.15 Results Defined by Equations 2.82 through 2.88 for Trajectories of Table 2.2	67

3.1 Double Inverted Pendulum	78
3.2 Value of I_1 , Case without Noise (LS vs. UKF)	78
3.3 Value of I_2 , Case without Noise (LS vs. UKF)	79
3.4 Value of m_1 , Case without Noise (LS vs. UKF)	79
3.5 Value of m_2 , Case without Noise (LS vs. UKF)	80
3.6 Value of M , Case without Noise (LS vs. UKF)	80
3.7 Value of I_1 , Case with Noise (LS vs. UKF)	81
3.8 Value of I_2 , Case with Noise (LS vs. UKF)	81
3.9 Value of m_1 , Case with Noise (LS vs. UKF)	82
3.10 Value of m_2 , Case with Noise (LS vs. UKF)	82
3.11 Value of M , Case with Noise (LS vs. UKF)	83
4.1 Single Strut Friction Test [37]	105
4.2 m_e Value (Force with Friction)	105
4.3 m_e Value (Force without Friction)	106
4.4 m_b Value (Force with Friction)	106
4.5 m_b Value (Force without Friction)	107
4.6 m_p Value (Force with Friction)	107
4.7 m_p Value (Force without Friction)	108
4.8 Mass Values (X-direction without Friction)	108
4.9 Mass Values (XY-direction without Friction)	109
4.10 Mass Values (Z-direction without Friction)	109
4.11 Mass Values (FF case without Friction)	110
4.12 Mass Values (X-direction with Friction)	110
4.13 Mass Values (XY-direction with Friction)	111

4.14 Mass Values (Z-direction with Friction).....	111
4.15 Mass Values (FF case with Friction)	112
4.16 m_e Value (Energy without Friction).....	112
4.17 m_b Value (Energy without Friction)	113
4.18 m_p Value (Energy without Friction)	113
4.19 Mass Values (X-direction: Energy-Based without Friction)	114
4.20 Mass Values (XY-direction: Energy-Based without Friction)	114
4.21 Mass Values (Z-direction: Energy-Based without Friction).....	115
4.22 Data Set Collection Type (Original, Spreading and Randomizing)	115
4.23 m_e Value (FA X WO – Original/Random/Spread)	116
4.24 m_b Value (FA X WO – Original/Random/Spread)	116
4.25 m_p Value (FA X WO – Original/Random/Spread)	117
4.26 m_e Value (FA Z WO – Original/Random/Spread).....	117
4.27 m_b Value (FA Z WO – Original/Random/Spread)	118
4.28 m_p Value (FA Z WO – Original/Random/Spread)	118
4.29 m_e Value (FA XY WO – Original/Random/Spread)	119
4.30 m_b Value (FA XY WO – Original/Random/Spread)	119
4.31 m_p Value (FA XY WO – Original/Random/Spread)	120
4.32 m_e Value (FF WO – Original/Random/Spread).....	120
4.33 m_b Value (FF WO – Original/Random/Spread).....	121
4.34 m_p Value (FF WO – Original/Random/Spread).....	121
4.35 Mass Value (X-direction, Random).....	122
4.36 Mass Value (XY-direction, Random)	122
4.37 Mass Value (Z-direction, Random)	123
4.38 Mass Value (FF case, Random)	123

4.39 Mass Value (X-direction, Spread)	124
4.40 Mass Value (XY-direction, Spread)	124
4.41 Mass Value (Z-direction, Spread).....	125
4.42 Mass Value (FF case, Spread)	125
4.43 m_e Value (0, 1 and 5 % Noise, Force-Based Model).....	126
4.44 m_e Value (0 and 1 % Noise, Energy-Based Model)	126
5.1 Single Strut Testing.....	146
5.2 Value of m_e (Experimental Result).....	146
5.3 Value of m_b (Experimental Result).....	147
5.4 Value of m_p (Experimental Result).....	147
5.5 Friction (1,1) Parameter Value (Experimental Result)	148
5.6 Friction (1,2) Parameter Value (Experimental Result)	148
5.7 Friction (2,1) Parameter Value (Experimental Result)	149
5.8 Friction (2,2) Parameter Value (Experimental Result)	149
5.9 Friction (3,1) Parameter Value (Experimental Result)	150
5.10 Friction (3,2) Parameter Value (Experimental Result)	150
5.11 Friction (4,1) Parameter Value (Experimental Result)	151
5.12 Friction (4,2) Parameter Value (Experimental Result)	151
5.13 Friction (5,1) Parameter Value (Experimental Result)	152
5.14 Friction (5,2) Parameter Value (Experimental Result)	152
5.15 Friction (6,1) Parameter Value (Experimental Result)	153
5.16 Friction (6,2) Parameter Value (Experimental Result)	153
C.1 m_b Value (0, 1 and 5 % Noise, Force-Based Modeling).....	171
C.2 m_p Value (0, 1 and 5 % Noise, Force-Based Modeling).....	172
C.3 m_b Value (0 and 1 % Noise, Energy-Based Modeling).....	172

C.4 m_p Value (0 and 1 % Noise, Energy-Based Modeling).....	173
C.5 m_e Value (0 % Noise, Force-Based vs. Energy-Based Modeling).....	173
C.6 m_b Value (0 % Noise, Force-Based vs. Energy-Based Modeling).....	174
C.7 m_p Value (0 % Noise, Force-Based vs. Energy-Based Modeling).....	174
C.8 m_e Value (1 % Noise, Force-Based vs. Energy-Based Modeling).....	175
C.9 m_b Value (1 % Noise, Force-Based vs. Energy-Based Modeling).....	175
C.10 m_p Value (1 % Noise, Force-Based vs. Energy-Based Modeling).....	176

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To date, very little research on the dynamics of parallel kinematic mechanisms/machines (PKMs) exists. Of the work available in the literature, most have been for generating the equations of motion, inverse dynamics, and structural dynamics. Insight to the structure and properties of PKM dynamics, effects of strut inertias, dynamics interactions with various kinematics error sources, formulation of dynamic performance indices, and dynamic balancing for PKMs have only recently been provided. Due to the above research or lack thereof has created an increasing awareness of the dynamic needs. Therefore, the thrust of the work presented in this dissertation is to establish a dynamic parameter identification approach for parallel kinematic machines. This approach is to acquire parameter values necessary for modeling the dynamics of parallel kinematic machines, e.g., mass, inertia, and friction.

To determine the best approach, several identification methods were compared and evaluated in terms of accuracy, convergence, ability to handle noisy data, and ease of implementation. The main comparison studies were conducted between the least squares

(LS) method and the unscented Kalman filter (UKF) method. To illustrate the superiority of the unscented Kalman filter (method of preference), both the LS and UKF methods were applied to the inverted double pendulum case. Using results found in the literature, the extended Kalman filter capabilities were compared to these two methods, further substantiating the superiority of UKF method for nonlinear systems. The comparisons were simulation-based. In addition, force-based and energy-based modeling methods are compared to determine if there is any benefit other than reduced modeling effort in deriving the theoretical model. A significant improvement using energy-based method was not generally realized. To experimentally validate the capability and the implementation of the UKF method for dynamic parameter identification, the University of Florida Space, Automation and Manufacturing Mechanisms Laboratory's PKM dynamic parameters were identified.

The simulated and experimentally validated results show that the unscented Kalman filter performs well in identifying the system parameters, eliminating the need for linearization. This merit corresponds to the significant reduction in modeling effort required to generate the basic algorithm used for the system identification. The overall impact to technology is provision of a parameter identification method that yields improved rigid body dynamics models of parallel kinematic mechanisms. This will lead to the ability to introduce advanced controllers that exploit the model dynamics to improve system performance. In addition, the presented method sets the foundation for formulating an online tuning algorithm.

NOMENCLATURE

$(\mathbf{a}_B)$ absolute acceleration of the center of mass of the body 'B' of strut in inertial frame {W} coordinates.

$\mathbf{a}_c$ absolute acceleration of the center of the mass of the moving platform in inertial frame {W} coordinates.

$(\mathbf{a}_E)$ absolute acceleration of the center of mass of the body 'E' of strut in inertial frame {W} coordinates.

b_i base platform-strut joint for joint i ; $i=1,6$.

d_k measured (desired) output

$\det[\]$ determinant of matrix []

e_i reference residual i .

F double pendulum example's controller input force (Chapter 3)

$\mathbf{F}_a$ 6-component vector $[F_{a_1}, F_{a_2}, \dots, F_{a_6}]^T$ of the six actuator forces $(F_{a_i})_i$, $i=1,6$ [units: (N, N, N, N, N, N)^T]

{F} coordinate system (X_F, Y_F, Z_F): frames with the same orientation as frame {W} but with origin located at the center of mass of the moving platform

$\begin{bmatrix} \mathbf{F}_{cen} \\ \mathbf{T}_{cen} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{cen}^T & \mathbf{T}_{cen}^T \end{bmatrix}^T$ 6-component vector of the (forces, moments) contribution

to $\mathbf{J} \mathbf{F}_a$ wrench due to the centripetal type accelerations of the strut masses [units:

(N, N, N, N-m, N-m, N-m)^T], defined in the inertial frame {W}.

$$\begin{bmatrix} \mathbf{F}_{cor} \\ \mathbf{T}_{cor} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{cor}^T & \mathbf{T}_{cor}^T \end{bmatrix}^T \quad \text{6-component vector of the (forces, moments) contribution}$$

to $\mathbf{J F}_a$ wrench due to the Coriolis type accelerations of the strut masses [units: (N, N, N, N-m, N-m, N-m)^T], defined in the inertial frame {W}.

$$\begin{bmatrix} \mathbf{F}_{ext} \\ \mathbf{T}_{ext} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{ext}^T & \mathbf{T}_{ext}^T \end{bmatrix}^T \quad \text{6-component vector of the (forces, moments) due to the}$$

externally applied forces and moments acting on the PKM [units: (N, N, N, N-m, N-m, N-m)^T], defined in the inertial frame {W}.

$$\begin{bmatrix} \mathbf{F}_{frict} \\ \mathbf{T}_{frict} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{frict}^T & \mathbf{T}_{frict}^T \end{bmatrix}^T \quad \text{6-component vector of the (forces, moments) contribution}$$

to $\mathbf{J F}_a$ wrench due to the joint friction [units: (N, N, N, N-m, N-m, N-m)^T], defined in the inertial frame {W}.

$$\begin{bmatrix} \mathbf{F}_{grp} \\ \mathbf{T}_{grp} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{grp}^T & \mathbf{T}_{grp}^T \end{bmatrix}^T \quad \text{6-component vector of the (forces, moments) contribution}$$

to $\mathbf{J F}_a$ wrench due to gravitational acceleration acting on the platform mass [units: (N, N, N, N-m, N-m, N-m)^T], defined in the inertial frame {W}.

$$\begin{bmatrix} \mathbf{F}_{grs} \\ \mathbf{T}_{grs} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{grs}^T & \mathbf{T}_{grs}^T \end{bmatrix}^T \quad \text{6-component vector of the (forces, moments) contribution}$$

to $\mathbf{J F}_a$ wrench due to gravitational acceleration acting on the strut masses [units: (N, N, N, N-m, N-m, N-m)^T], defined in the inertial frame {W}.

$$\begin{bmatrix} \mathbf{F}_{pm} \\ \mathbf{T}_{pm} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix}^T \quad \text{6-component vector of the (forces, moments) contribution}$$

to $\mathbf{J F}_a$ wrench due to the linear and angular accelerations of the platform mass

[units: (N, N, N, N-m, N-m, N-m)^T], defined in the inertial frame {W}.

$$\begin{bmatrix} \mathbf{F}_{sm} \\ \mathbf{T}_{sm} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix}^T \quad \text{6-component vector of the (forces, moments) contribution}$$

to $\mathbf{J F}_a$ wrench due to the linear and angular accelerations of the strut mass [units:

(N, N, N, N-m, N-m, N-m)^T], defined in the inertial frame {W}.

$$\begin{bmatrix} \mathbf{F}_{umdl} \\ \mathbf{T}_{umdl} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{umdl}^T & \mathbf{T}_{umdl}^T \end{bmatrix}^T \quad \text{6-component vector of the (forces, moments) contribution}$$

to $\mathbf{J F}_a$ wrench due to un-modeled dynamics [units: (N, N, N, N-m, N-m, N-m)^T], defined in the inertial frame {W}.

$(F_{\zeta_i})_i$ magnitude and sense of the translational friction force for friction in strut i,

direction along $(\hat{\mathbf{s}}_3)_i$; i=1,6.

$[\mathbf{I}]$ a 3x3 identity matrix

(I_{xx}) , (I_{yy}) , and (I_{zz}) principal mass moment of inertias of the platform (kg-m²), defined about the center of mass of platform with principal axes $\{\hat{\mathbf{s}}_1, \hat{\mathbf{s}}_2, \hat{\mathbf{s}}_3\}_i$, respectively.

$(I_{xxb})_i$, $(I_{yyb})_i$, and $(I_{zzb})_i$ principal mass moment of inertias of the nonextendable portion of the strut (kg-m²), defined about its center of mass with principal axes $\{\hat{\mathbf{s}}_2, \hat{\mathbf{s}}_3, \hat{\mathbf{s}}_{23}\}_i$, respectively. One set for each strut i.

(Note, I_{yyb} is moment of inertia about $\hat{\mathbf{s}}_1$ axis and $I_{xxb}=I_{zzb}$ are about the corresponding orthogonal axis to $\hat{\mathbf{s}}_1$)

$(I_{xxc})_i, (I_{yye})_i$, and $(I_{zzc})_i$ principal mass moment of inertias of the extendable portion of the strut ($\text{kg}\cdot\text{m}^2$) defined about its center of mass with principal axes

$\{\hat{s}_2, \hat{s}_3, \hat{s}_{23}\}_i$, respectively. One set for each strut i .

(Note, I_{yye} is moment of inertia about $\hat{s}_3$ axis and $I_{xxc}=I_{zzc}$ are about the corresponding orthogonal axis to $\hat{s}_3$)

$\mathbf{J}, \mathbf{J}^{-1}$ (6x6) pseudo-Jacobian matrix and inverse of the pseudo-Jacobian matrix, respectively

$J_{LS,k}$ Jacobian matrix used in the least squares method

$(L\hat{s}_3)_i$ vector denoting i^{th} strut length L_i along strut axis $(\hat{s}_3)_i$; one for each strut; $i=1,6$.

$(\dot{L}\hat{s}_3)_i$ vector denoting the time rate of change of the strut length $(\dot{L})_i$ along strut axis $(\hat{s}_3)_i$; one for each strut; $i=1,6$.

$(\ddot{L}\hat{s}_3)_i$ acceleration component of strut that is aligned with unit vector $(\hat{s}_3)_i$; one for each strut; $i=1,6$, a vector denoting the 2nd derivative of L_i .

$(L_B\hat{s}_3)_i$ a 3-component vector denoting the location of the center of mass of the composite nonextendable part of strut i from center of the base platform-strut joint b_i along $(\hat{s}_3)_i$ axis; $i=1,6$.

$(\dot{L}_B\hat{s}_3)_i$ time rate of change of the distance $(L_B)_i$ along strut axis $(\hat{s}_3)_i$; $i=1,6$.

Note, $\dot{L}_B=0$ since the L_B does not change in time.

$(\ddot{L}_B)_i$ time rate of change of $\dot{L}_B$, Note, $\ddot{L}_B=0$ since the $\dot{L}_B$ does not change in time

$(L_e \hat{s}_3)_i$ a 3-component vector denoting the location of the center of mass of the composite extendable part of strut i measured from the center of the base joint b_i along $(\hat{s}_3)_i$ axis; $i=1,6$.

$(\dot{L}_e \hat{s}_3)_i$ time rate of change of $(L_e)_i$ along strut axis $(\hat{s}_3)_i$; $i=1,6$.

$(\dot{L}_e)_i$ time rate of change of $(L_e)_i$; $i=1,6$.

l_1, m_1 length and mass of the bottom pendulum (link 1), respectively (Chapter 3)

l_2, m_2 length and mass of the top pendulum (link 2), respectively (Chapter 3)

M mass of the cart (Chapter 3)

$[M_p]$ mass matrix relating to $[F_{pm}^T, T_{pm}^T]^T$

$(m_p)_i, (I_p)_i$ lumped mass and inertia tensor of the moving platform, respectively.

Center of mass located at origin of $\{P\}$ frame

$(m_b)_i, (I_b)_i$ lumped mass and inertia tensor of the nonextendable portion of the strut i ; $i=1,6$, respectively.

$(m_e)_i, (I_e)_i$ lumped mass and inertia tensor of the extendable portion of the strut i ; $i=1,6$, respectively.

$\{P\}$ coordinate system (X_P, Y_P, Z_P) : frame rigidly attached to the moving platform and used to define the absolute orientation of the moving platform, origin is located of the platform's center of mass.

P_i moving platform-strut joint for strut i ; $i=1,6$.

$\mathbf{q} = [x_c, y_c, z_c, \phi, \theta, \psi]^T$ 6-component vector of the generalized coordinates of the system, the first 3-components are the $\{W\}$ coordinates of the absolute position of

the moving platform's center of mass and the last 3 components are the Euler angles defining the absolute orientation.

$\dot{\mathbf{q}} = [\dot{x}_c, \dot{y}_c, \dot{z}_c, \dot{\phi}, \dot{\theta}, \dot{\psi}]^T$ 6-component vector of the generalized velocity, first derivative with respect to time of the generalized coordinates

${}^W \mathbf{r}_c = (x_c, y_c, z_c)^T$ 3-component position vector of the center of mass of the platform specified in {W} coordinate system [units: (m, m, m)^T].

$({}^W \mathbf{r}_b)_i$ 3-component position vector of the ith base -strut joint location (b_i) defined relative to the {W} origin; i=1,6.

$({}^W \mathbf{r}_s)_i$ 3-component position vector of the center of mass of the nonextendable part of the ith strut defined relative to the {W} origin; i=1,6.

$({}^W \mathbf{r}_p)_i$ 3-component position vector of the ith moving platform-strut joint location (P_i) defined relative to the {W} origin; i=1,6

$({}^W \mathbf{r}_e)_i$ 3-component position vector of the center of mass of the extendable part of the ith strut defined relative to the {W} origin; i=1,6.

$(\mathbf{r}_{pc})_i$ 3-component position vector of the ith moving platform-strut joint location (P_i) defined relative to the platform's center of mass (C), specified in {W} coordinates; i=1,6 [units: (m, m, m)^T]

${}^W \mathbf{R}_p$ 3 × 3 orthonormal rotational matrix defining the orientation of frame {P} relative to frame {W} coordinates, note {W} and {F} are parallel frames for all time.

${}^F \mathbf{R}_p$ 3 × 3 orthonormal rotational matrix defining the orientation of frame {P} relative to frame {F} coordinates.

${}^W\mathbf{R}_F$ 3×3 orthonormal rotational matrix defining the orientation of frame $\{F\}$ relative to frame $\{W\}$ coordinates.

$\{\mathbf{S}_i\}$ orthogonal coordinate system $\{\hat{\mathbf{s}}_2, \hat{\mathbf{s}}_3, \hat{\mathbf{s}}_{23}\}_i$ affixed to strut i with origin at joint b_i . $\{\hat{\mathbf{s}}_2, \hat{\mathbf{s}}_3, \hat{\mathbf{s}}_{23}\}_i$ are specified in $\{W\}$ coordinate system.

$(\hat{\mathbf{s}}_1)_i$ 3-component unit vector defining the first joint axis of the Hooke joint, parallel to x-axis of $\{W\}$ coordinate system specified in $\{W\}$ coordinates; $i=1,6$; total of six $(\hat{\mathbf{s}}_1)_i$ unit vectors, one for each strut.

$(\hat{\mathbf{s}}_2)_i$ 3-component unit vector orthogonal to $(\hat{\mathbf{s}}_3)_i$ and $(\hat{\mathbf{s}}_1)_i$ vector specified in $\{W\}$ coordinates; $i=1,6$; total of six $(\hat{\mathbf{s}}_2)_i$ unit vectors, one for each strut.

$(\hat{\mathbf{s}}_{23})_i$ 3-component unit vector orthogonal to $(\hat{\mathbf{s}}_2)_i$ and $(\hat{\mathbf{s}}_3)_i$ vector, direction defined by $((\hat{\mathbf{s}}_2)_i \times (\hat{\mathbf{s}}_3)_i)$ specified in $\{W\}$ coordinates; $i=1,6$; total of six $(\hat{\mathbf{s}}_{23})_i$ unit vectors, one for each strut.

$(\hat{\mathbf{s}}_3)_i$ 3-component unit vector, aligned along the strut's longitudinal axis specified in $\{W\}$ coordinates; $i=1,6$; total of six $(\hat{\mathbf{s}}_3)_i$ unit vectors, one for each strut.

$(\)^T$ transpose of the $(\)$.

$(T_\phi)_i, (T_\phi)_i$ torsional friction in strut-platform joints (P_i and b_i), moving platform and base platform, respectively; direction of corresponding torque vector is aligned with relative angular velocity vector in the joint.

$\mathbf{U}$ 3×3 matrix transformation from the Euler angular rates to the inertia frame definition of the platform's absolute angular velocity

$(\mathbf{v}_B)_i$ absolute velocity of the center of mass of the nonextendable part of strut in inertial frame $\{W\}$ coordinates.

$\mathbf{v}_e$ absolute velocity of the center of the mass of the moving platform in inertial frame $\{W\}$ coordinates.

$(\mathbf{v}_E)_i$ absolute velocity of the center of mass of the extendable part of strut in inertial frame $\{W\}$ coordinates.

$\{W\}$ coordinate system (X_W, Y_W, Z_W) : a fixed, Cartesian inertial reference frame, frame of reference for all absolute vectors

w_k the parameter set to be identified at the k^{th} iteration step in its estimation (chapter 3)

x position of the cart in horizontal plane (Chapter 3)

$\dot{x}$ a velocity of the cart in horizontal plane (Chapter 3)

y_k updated output (calculated from measured states (x_k) and estimated parameters (w_k)) for k^{th} iteration step

z-x-z Euler platform frame $\{P\}$'s absolute orientation is defined relative to frame $\{F\}$ by ordered z-x-z Euler angular rotations: First ϕ rotation about frame $\{F\}$ axis Z , then θ rotation about new X axis, and finally rotation ψ about new Z axis (units: radians)

$(\alpha_B)_i$ absolute angular acceleration of body 'B' of strut defined in inertial frame $\{W\}$ coordinates, $(\alpha_B)_i = (\alpha_E)_i$ since $(\omega_B)_i = (\omega_E)_i$.

α_e absolute angular acceleration of the moving platform defined in inertial frame $\{W\}$ coordinates

$(\alpha_E)_i$ absolute angular acceleration of body 'E' of strut defined in inertial frame $\{W\}$ coordinates, $(\alpha_E)_i = (\alpha_B)_i$ since $(\omega_E)_i = (\omega_B)_i$.

- β 3-component vector consisting of the z-x-z Euler angles, defines the moving platform's absolute orientation.
- $\dot{\beta}$ 3-component vector consisting of the time rate of change of β , Euler angular rates for the moving platform.
- $\ddot{\beta}$ 3-component vector consisting of the time rate of change of $\dot{\beta}$.
- θ_1 absolute angular position of the bottom pendulum (link 1) (Chapter 3).
- $\dot{\theta}_1$ absolute angular velocity of the bottom pendulum (link 1) (Chapter 3).
- θ_2 absolute angular position of the top pendulum (link 2) (Chapter 3).
- $\dot{\theta}_2$ absolute angular velocity of the top pendulum (link 2) (Chapter 3).
- $(\omega_B)_i$ absolute angular velocity of body 'B' of strut defined in inertial frame {W} coordinates. Note, $(\omega_E)_i = (\omega_B)_i = (\omega_s)_i$ for link i expressed in frame {W} coordinates, angular velocities of bodies B_i and E_i which are constrained to rotate together due to prismatic joint connecting the two bodies.
- $\omega_c = [U] \dot{\beta}$ the absolute platform angular velocity defined in inertial frame {W} coordinates.
- $(\omega_E)_i$ absolute angular velocity of body 'E' of strut defined in inertial frame {W} coordinates. Note, $(\omega_E)_i = (\omega_B)_i = (\omega_s)_i$ for link i expressed in frame {W} coordinates, angular velocities of bodies B_i and E_i which are constrained to rotate together due to prismatic joint connecting the two bodies.
- $(\tilde{\omega}_s)_i$ absolute strut angular velocity defined in inertial frame {W},
- $$(\tilde{\omega}_s)_i = (\omega_{s_{23}})_i (\hat{s}_2)_i + (\omega_{s_{31}})_i (\hat{s}_3)_i + (\omega_{s_{23}})_i (\hat{s}_{23})_i$$

where

$(\omega_{s_{s_2}})_i$ angular component of strut i about $(\hat{s}_2)_i$ axis

$(\omega_{s_{s_3}})_i$ angular component of strut i about $(\hat{s}_3)_i$ axis

$(\omega_{s_{s_{23}}})_i$ angular component of strut i about $(\hat{s}_{23})_i$ axis

$(\omega_s)_i$ absolute strut angular velocity defined in inertial frame {W} with $(\omega_{s_{s_3}})_i=0$,

$$(\omega_s)_i = (\omega_{s_{s_2}})_i (\hat{s}_2)_i + (\omega_{s_{s_{23}}})_i (\hat{s}_{23})_i$$

CHAPTER 1 INTRODUCTION

1.1 History of Parallel Kinematic Machines

Although due in part to their anthropomorphic nature, serial manipulators comprised of open chain linkages are commonly used in industry. However, since the science of robotics has developed, interest in parallel kinematic mechanisms has grown so that many of the serial robots' actuation transmissions are actually comprised of closed chain linkages (the general class in which parallel kinematic mechanisms belong). The Stewart platform [1] was one of the first implementations of the parallel kinematic mechanism and has been widely adapted as an aircraft simulator (see Figure 1.1). Although it has been claimed by researchers that parallel kinematic mechanisms/machines (PKMs) have higher structural stiffness, and proportionally distribute the payload to the links, these attributes are highly dependent on their configuration design (e.g., Stewart-Gough, Delta-like, Triaglide, Linapod-like). Compared to serial manipulators, PKMs have the advantage of relatively simple inverse kinematic calculations yielding unique solutions, but have the disadvantage of the forward kinematics resulting in high order polynomials with multiple solutions.

Using application driven design methods, PKMs can be realized that exhibit sufficient structural stiffness and high speed motions for a wide variety of applications (e.g., motion simulations, assembly and packaging end-effectors, surgical, MEMS, entertainment). Limited usage of PKMs is largely due to small workspace, small regions of dexterity, variable end-effectors stiffness, non-unique solutions of forward kinematics,

workspace singularities, self-motions and uncomprehended dynamic performances.

During the last decade, there have been many debates on the ‘true’ attributes of PKMs and their applications, especially since their 1994 début as a new machine tool paradigm.

In the literature [2], it was described as follows.

The PKM suffers from a lack of comprehensive calibration techniques, from weak or not well-defined dynamics behavior, and from the need for complex control algorithms for the compensation of stiffness variations in the workspace.

To address the above issues and subsequently enable greater understanding and utilization, this dissertation’s research focuses on the dynamics. The next few sections include the brief summary of the research accomplishments and applications, which are now being implemented, followed by the proposed methodology, and research plan.

1.2 Types of PKM and Applications

Parallel mechanisms can be classified as 3 basic types:

- Variable link length (Stewart-Gough)
- Variable two link orientation (Delta-like)
- Fixed link length with variable base joint positions (Triaglide or Linapod-like)

These types of PKMs and their variants are commonly found in industry and research laboratories. The following highlights a few of the variants in design.

For a spherical parallel manipulator (SPM) designed to orient a rigid body in space, researchers used kinematic conditioning as a performance index to optimize the manipulator configuration [3, 4]. Figure 1.2 shows a sketch of the SPM. This manipulator could be used for the orientation of parabolic antennas, panels, and machine tool beds. Such a mechanism can also be applied as an orientation of the wrist for an industrial

robot. The research by Gosselin and Angles [3] includes the restriction of symmetry of the machine kinematics, workspace maximization, and isotropy of the machine. A discussion of the singularities was also included.

Many researchers proposed novel kinematic structures for a parallel manipulator with 6 DOF (Figure 1.3). Byun and Cho [5] showed a feasible real-time computation for all forward solutions of a 6 DOF parallel manipulator consisting of a base, platform, and 3-PPSP (P-Prismatic, S-Spherical) serial mechanisms. Their machine was to overcome the common drawbacks of the conventional six-degree of freedom parallel manipulators, which are complex forward computation, small workspace volume, and limited load capacity. However, the resulting PKMs are still constrained by a relatively small functional workspace. These PKM structures could be used as a motion base, which generally needs only a small workspace capacity. The authors [5] did not address workspace restrictions, singularity points, and the dynamics of the structure.

Other researchers have conceived an idea for a parallel manipulator (Turin parallel manipulator named TuPaMan, see Figure 1.4) [6]. This manipulator is composed of three articulated double parallelograms, three prismatic joints, three spherical joints, and a moving upper platform. The TuPaMan was strongly influenced by the Stewart platform and can be used as a manipulator wrist or arm. The restrictions for TuPaMan are a function of the size of the device, using the overall dimensions and the area of the middle cross-section of the workspace as synthetic performance indices. Oh [7] used linear motors to extend the workspace. He presents a method to overcome three constraints: strut collision, base puck collision, and singularity considerations.

1.3 PKM Kinematics

The PKM kinematics has been extensively investigated for the past few decades. Much of kinematic research focus has been concentrated on solving the forward kinematics problem. The traditional Stewart Platform Mechanism has 40 mathematically achievable configurations in the solution set. Duffy *et al.* [8] reduced the forward kinematic solutions to eight for a special 6-6 PKM.

1.4 PKM Dynamics and Controls

To date, not much research on the dynamics of the PKM has been done. The dynamic equations of motion have been derived for the generalized Stewart platforms using Lagrangian, Newton-Euler, and Kane's method. Of the work available in the literature, a sampling of researchers who investigated modeling, dynamics and/or parameter identification in the robotics community are: [9-23]. Sestieri [9] derived a series of simple linear problems from the non-linear problem, and solved them with a frequency-response function (FRF). Tinone and Aoshima [10] used a repetitive controller in order to reduce bias from measurement error. Lin [16] used MLC (minimal linear combinations) for avoiding the difficulties to estimate the parameters. The MLC was used for estimating the parameters in the serial manipulators, but it can also be used in the parallel manipulator. Lin described MLC excluding the friction parameters. Further insight into PKM and serial robot dynamics and identification has been provided in the following: vibration analysis [24], trajectory analysis [25], force analysis [26] accuracy analysis [27], forward analysis [28], stiffness analysis [29], and parameter estimation [30-33]. Available in the literature, only one reference [33] attempted parameter identification of PKM's rigid body dynamics.

1.5 Prior Work within SAMM Lab

The following is a summary of the research on PKMs conducted within the Space, Automation and Manufacturing Mechanisms (SAMM) Laboratory. Hardage [34] developed a methodology for identifying the structural dynamic parameters of PKMs validated using experimental data from the Hexel PKM located at Sandia National Laboratory. He used component mode synthesis and Lagrangian techniques to derive the equations of vibratory motion. Modal testing was performed to measure the stiffness and frequency of the Sandia National Laboratories Hexel PKM. His results demonstrated that the Hexel machine could not be expected to achieve high structural performance nor be widely adopted as a machine tool.

Johnston's research [35] was to develop dynamic performance indices for a parallel manipulator. In his work, Kane's method was used to establish the equations of dynamic motion. To eliminate mixed units in this metric, he modified the energy expression to be in terms of linear velocities of the platform only. The performance indices clearly demonstrated that to fully characterize the dynamic performance of the machine, kinematics based indices are not sufficient. This was further substantiated by Walker's research on studying the effects of kinematic errors on dynamic performance.

Register [36] characterized the effects of friction using a parameter estimation algorithm (extended Kalman filter). In his work, friction forces were shown to be dominant over inertial and gravitational forces in linear and angular motions of the struts. Estimating the friction parameter for the single joint strut was achieved within 10 percent using the estimation algorithm, however, the identification of friction parameters in multiple joint types was not addressed in his thesis. Scott Shamblin [37] conducted

further analysis and experimental identification of the individual struts and the assembled PKM and again was faced with the challenges of high friction.

1.6 System Identification Methods

System identification methods are well known and utilized by researchers working in the areas of dynamics, controls, and estimation theory. System identification can be categorized into state, parameter and dual estimation. The parameter estimation method is used for approximating the unknown parameters of the system from measured data which contains the known system input and output variables. There are many methods existing in parameter estimation, however due to its mathematical simplicity, a commonly used method is least squares. The least squares method is widely used to find system parameter and state estimation. Least squares methods, however, exhibits convergence problems in the presence of noise and lacks the ability to effectively handle nonlinearities in system identification. This has lead to the development of various mutations of least squares [23]. Of these mutations, a recent method gaining popularity is the extended Kalman filter. The most efficient way to identify nonlinear system parameters, however, is the unscented Kalman filter method (UKF) [38]. For this dissertation's research, the unscented Kalman filter method is selected as the method of choice for dealing with the dynamic complexity of PKMs. The major advantage for the UKF is that it does not require the calculation of Jacobian and Hessian matrices. This approach will be shown in Chapter 4 to yield faster and more convenient solutions for the nonlinear equations of the PKM. An example for comparing the least squares method and the unscented Kalman filter method is provided in Chapter 3, demonstrating the effectiveness of the UKF method.

1.7 Problem Statement

When a system has unknown inertial parameters that are difficult to identify by simple physical measurements of the components, a parameter estimation method can resolve the variables using commanded inputs and measured output response values. The main goal of this dissertation is to develop a methodology for identifying the system dynamic model of PKMs through estimating the inertial parameters. The important characteristic of the inertial parameters of the machine, in general, is that those values are invariant in time. In other words, when changing the state values of system over time, the inertial parameters stay the same regardless of what the input and output values.

The outline of the procedure established to obtain the inertial parameters presented in this dissertation is as follows:

First, formulate a model base by deriving the equations of motion. The theoretical background relating to the PKM dynamics and estimation method is shown (Chapter 2). Second, analyze and compare candidate estimation techniques. Least squares (LS) method, extended Kalman filter (EKF), and unscented Kalman filter (UKF) were selected for comparisons as representing the spectrum of identification methods (Chapter 3). For this comparison, the inverted double pendulum example was selected. Simulation results were also generated for exploring both force-based modeling and energy-based modeling as a model basis for implementing the preferred estimation method (UKF, Chapter 4). Third, experimental validation of the capability and the implementation of the UKF estimation method is demonstrated in Chapter 5. The measured PKM input and output responses from the experiment were employed in the estimation program. Finally, the

validation and reconciliation of the resulting parameter identification methodology were completed.

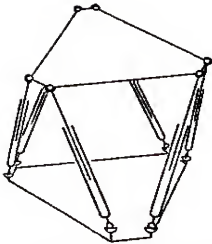


Figure 1.1 Stewart Platform [1]

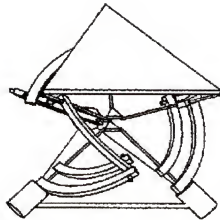


Figure 1.2 Spherical Parallel Manipulator [3]

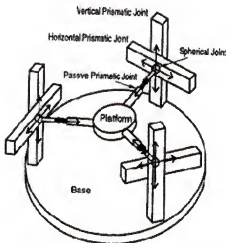


Figure 1.3 Novel 6-DOF Parallel Machine [5]

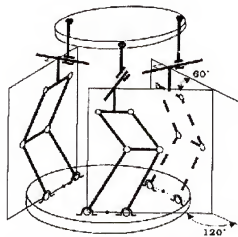


Figure 1.4 Turin Parallel Manipulator (TuPaMan) [6]

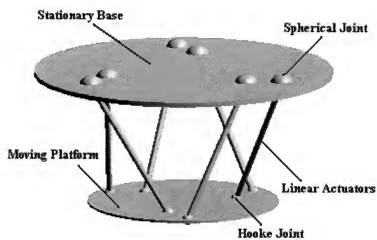


Figure 1.5 Generalized Stewart Platform Schematic [36]

CHAPTER 2

THEORETICAL BACKGROUND

Each of the parameter identification methods considered in this dissertation require as a model-base the theoretical model of the expected system dynamics. In the derivation of the theoretical model, two different approaches were taken in formulating a model-base. One approach uses the system's equations of motion, referred to as the force-based model. The other approach uses the expressions from the system's energy equations, referred to as the energy-based model. The same force-based model also serves as a theoretical representation of the physical system in the simulated identification implementation studies and other supporting studies. In this chapter, the theoretical modeling of the general class of Stewart-Gough parallel kinematic mechanisms (PKM) is presented with supplemented details given in Appendices A and B. The chapter continues with presentation of the theoretical details of three parameter identification methods. Simulated and experimental implementations of these methods are presented in Chapters 3, 4 and 5. The chapter closes with a discussion of implementation issues and preliminary characterization of the expected dynamic parameter dominance, the ease in identifying various terms, and effects of limited and/or indirect sensing. While the dissertation results are limited to the Stewart-Gough class of PKMs, it should be noted that the methodology developed in this dissertation and the issues addressed are directly applicable to PKMs, in general.

2.1 System Description and Model

In this dissertation, the Stewart-Gough PKM type is assumed. This type of PKM has six extendable struts (linear actuators) that provide 6-degrees of freedom (d.o.f.) to a moving platform relative to a base platform, refer to schematic in Figure 2.1. Figure 2.2 shows a physical diagram of a representative strut, and figure 2.3 is its corresponding lumped mass model where details are provided in the section on equations of motion. The following describes three PKM's that are a part of the Stewart-Gough family of PKM's. Each consists of different mass distribution and special joint arrangements where the strut design is generally chosen by the designer to minimize computation and to facilitate actuation. In section 2.4, these three PKM configuration variations were used to characterize the expected parameter dynamics and to delineate potential implementation issues.

2.1.1 System Description

2.1.1.1 UF-PKM

Figure 2.4 shows the Special 6-6 PKM [8] which is the design type of the University of Florida – Space, Automation and Manufacturing Mechanisms (UF-SAMM) Laboratory's testbed's PKM, a subset of Stewart-Gough PKMs. The special 6-6 PKM consists of a moving platform with its 6 degrees of motion actuated via six extendable struts. The special 6-6 PKM has a unique telescoping strut arrangement such that both the platform and base consist of three sets of three joints lying on lines that form the sides of a triangle. It is this joint configuration that yields reduced computation requirement for the forward kinematic solutions compared to that required of the general Stewart-Gough PKM as well as allows for simpler physical joint designs. As shown in Figure 2.2, each strut consists of a motor, encoder, brake, spherical joint, ball screw,

Hooke joint (also known as universal joint or cardan joint), hollow ball spline/nut, and outer tube. Each of the struts (linear actuators) is connected to the stationary base platform through a large hollow spherical joint. This connection allows the strut to rotate through space as its length is varied. A servomotor, whose armature is connected to a ball screw that passes through the center of the hollow spherical joint, drives the strut. To prevent rotation about the servomotor axis, the combination of a hollow ball spline and a Hooke joint is used as the connection between the moving platform and the strut.

2.1.1.2 SNL-PKM

Figure 2.4 (b) presents the Sandia National Laboratory's Hexel parallel machine tool. Still a Stewart-Gough type PKM, its configuration differs in that the motor and ball nut of each strut are concentric with the base spherical joint and the ball screw passes through the base joint connecting directly to the moving platform, thus eliminating the need for a hollow ball spline and outer tube. The Hooke joints are replaced with bifurcated ball joints that allow for concentric spherical joint motion between two struts and the moving platform. The joints are arranged between the struts and the base and moving platforms as illustrated in Figure 2.4 (b). Between the moving platform and struts, the joint arrangements form an equilateral triangle.

2.1.1.3 FORD-PKM

Figure 2.4 (c) is of the Ford PKM used for full 6-degrees of freedom vehicle simulation with the driver in-the-loop control. The Ford PKM has an approximate octahedral layout similar to the Hexel base platform. The platform joint configuration(s) are arranged in close pairs for both the base and moving platforms. This PKM is powered by hydraulics, i.e., each strut is essentially a hydraulic cylinder with its ends connected to the base and moving platforms. Both the Ford and UF PKMs have

relatively short struts in relation to their base joint spread, but are totally different in terms of the mass distribution. I.e., the moving platform of the UF PKM has relative small inertia compared to the inertia of struts, while the Ford PKM has the majority of its inertia in the moving platform.

2.1.2 Lumped Mass Parameter Model

The physical system has three major components to be categorized: the stationary base, the strut and the moving platform. The stationary base serves as the inertial reference frame for the system. The moving platform has six degrees of freedom and consists of a single mass. It has a mass m_p and inertia tensor $\mathbf{I}_p$. The last major component is the strut. Since the actuators only produce translational motion along the longitudinal axis of the struts, the actuating component is modeled as a linear force applied across a prismatic joint. For the UF-PKM, the struts are connected between the stationary base platform and a moving platform with a spherical joint and a Hooke joint, respectively. However, without loss of generality and for modeling purposes, each strut can be theoretical modeled as forming a kinematic chain (H-P-S) which consists of a Hooke joint, a prismatic joint (representing the linear actuation), and a spherical joint. Both kinematic chains have the same kinematic degrees of freedom. Figures 2.2 and 2.3 show the physical diagram of a single strut for the UF PKM and the corresponding lumped mass model. For Body B_i , the $(m_b)_i$ and $(\mathbf{I}_b)_i$ are respectively the composite mass and inertia tensor for all the components in the physical model that do not extend (or translate along strut axis $(\hat{s}_3)_i$) as the strut (linear actuator) changes in length. For the UF-PKM, this consists of an outer tube, ball screw, ball of the spherical joint, motor, encoder and brake. The strut length is defined by the distance between center of the

moving platform-strut joint (P_i) and base-platform-strut joint (b_i). For Body E_i , the $(m_e)_i$ and $(I_e)_i$ are respectively the composite mass and inertia for the remaining components that exhibit a translational motion along the $(\hat{s}_3)_i$ axis. For the UF-PKM, this consists of a ball nut, hollow ball spline, and Hooke joint. The actuator's force $(F_a)_i$ can be modeled as a force exerted between these two lumped mass entities. The joint friction within the kinematic chain can be modeled as both translational and torsional friction, $(F_{\zeta})_i$, $(T_{\zeta_h})_i$, and $(T_{\zeta_b})_i$, respectively. The quantities $(L_B)_i$, $(L_E)_i$, and L_i denote respectively the position along strut axis $(\hat{s}_3)_i$ relative to point (b_i) of the lumped masses, $(m_e)_i$ and $(m_b)_i$, center of mass and L_i is the instantaneous length of strut i . In reiterating the above and illustrated in Figure 2.3, the strut masses are lumped into two parts, Body E_i (m_e , I_e) $_i$ and Body B_i (m_b , I_b) $_i$, which both experience rotational motion relative to the base joint, i.e., rotate with $(\hat{s}_3)_i$ axis. In addition, Body E_i experiences translational motion along $(\hat{s}_3)_i$ where Body B_i does not. Since Body E_i translates freely in the direction of the strut axis, the location of its center of mass is located at a variable distance measured from the base platform-strut joint location (b_i) along the strut axis $(\hat{s}_3)_i$. As stated earlier, this distance is denoted by $(L_E)_i$. The velocity and acceleration of Body E_i can be expressed in terms of the linear velocity and acceleration of the extendable part of the strut in the direction of the strut axis and the strut angular velocity and acceleration about axes defined in the base joint. Body B_i has a center of mass that is located a fixed distance $(L_B)_i$ from the base joint (b_i) and is measured in the direction of the strut axis. Since this distance is fixed, the velocity and acceleration of Body B_i can be expressed in terms of the strut's angular velocity and acceleration about the axes defined

in the base joint (b_i). To summarize, the total number of the lumped masses in the Stewart-Gough PKM model are thirteen: two for each strut and one for the moving platform.

2.1.3 Coordinate System Assignment

In the analysis of kinematics and dynamics, it is typical that a number of coordinate systems are defined to describe the motion of bodies. To formulate the dynamic equations in this dissertation, three types of coordinate systems are defined: global (the inertial reference frame), strut (S-coordinate system), and moving platform coordinate systems (translational and rotational coordinate system).

2.1.3.1 Global and Moving Platform Coordinate Systems

The global (inertial) coordinate system is denoted as $\{W\}$ coordinate system. It is a fixed, Cartesian inertial reference frame. This coordinate system can be located arbitrarily, however, for the PKMs considered, the $\{W\}$ coordinate system is fixed at the geometric center location of the stationary base platform as shown in Figure 2.5. The moving platform coordinate system consists of two frames: a translational and rotational coordinate system. The translation frame is called the $\{F\}$ coordinate system and the rotation frame is called the $\{P\}$ coordinate system. As in Figure 2.5, the origin of the $\{F\}$ coordinate system moves with the center of mass location of the moving platform but the orientation of the $\{F\}$ frame remains parallel with the $\{W\}$ frame for all time. It shares the same origin with the $\{P\}$ coordinate system. The $\{P\}$ frame is affixed to the moving platform, where its absolute orientation and position is used to describe the motion of the moving platform relative to the $\{W\}$ frame. The $\{F\}$ frame is used as an intermediate step for obtaining the $\{P\}$ frame's absolute position and orientation. The $\{P\}$ frame orientation is arbitrary, but is typically selected to have its axes aligned with the principal

axes of inertia (as done in this dissertation's examples). To avoid singularities in the computation the axes are also selected to have specific orientations when the platform is in its 'home' position. This is explained below in section 2.1.1.2 c.

The naming convention used for denoting the center location of the strut joint on the base and moving platform as well as the center of mass location of the moving platform are shown in Figure 2.6. All joints on the stationary base are labeled point b_i where $i=1,6$ for six struts. Their locations are defined in the $\{W\}$ coordinates (e.g., ${}^w b_i$). On the moving platform, the locations of the joints are denoted as P_i . The displacement from the origin of the $\{F\}$ or $\{P\}$ coordinate system to point P_i is called vector $({}^w \mathbf{r}_{p_i})_i$, i.e., location of P_i relative to the platform's center of mass, C , expressed in $\{W\}$ coordinates.

2.1.3.2 Strut Coordinate System

The strut coordinate system is denoted as the $\{S\}_i$ coordinate system (refer to Figure 2.7 which shows the theoretical Hooke joint model for the base joint b_i). The $(\hat{\mathbf{s}}_1)_i$ unit vector is used to denote the first axis in the Hooke joint. The angular rate about this axis is defined as $(\omega_{s_1})_i$. Used as a reference to define other vectors and without loss of generality, the direction of the $(\hat{\mathbf{s}}_1)_i$ vector is arbitrarily defined to be parallel to the X_W -direction of $\{W\}$ coordinate system. I.e., the X-axis of the X-Y-Z coordinate frame in Figure 2.7 theoretically remains parallel to the $\{W\}$ coordinate system for all time. The $(\hat{\mathbf{s}}_3)_i$ unit vector is defined as the unit vector along the line of action between joint P_i and b_i , which is calculated as

$$(\hat{\mathbf{s}}_3)_i = \frac{({}^w \mathbf{r}_p)_i - ({}^w \mathbf{r}_b)_i}{\|({}^w \mathbf{r}_p)_i - ({}^w \mathbf{r}_b)_i\|} \quad (2.1)$$

Note, the subscripts i denote i^{th} strut where $i=1,6$ for the PKM's under consideration.

The $(\hat{s}_2)_i$ unit vector is formed as the cross product of $(\hat{s}_1)_i$ and $(\hat{s}_3)_i$ divide with the magnitude of the cross product of $(\hat{s}_1)_i$ and $(\hat{s}_3)_i$. The $(\hat{s}_{23})_i$ unit vector is defined as the cross product of $(\hat{s}_2)_i$ and $(\hat{s}_3)_i$. Thus, the three orthogonal unit vectors form the strut coordinate system expressed by equations (2.1) through (2.3) in $\{W\}$ frame coordinates.

$$(\hat{s}_1)_i = [1 \ 0 \ 0]^T \text{ and } (\hat{s}_2)_i = \frac{(\hat{s}_1 \times \hat{s}_3)_i}{\|\hat{s}_1 \times \hat{s}_3\|_i} \quad (2.2)$$

$$(\hat{s}_{23})_i = (\hat{s}_2 \times \hat{s}_3)_i \quad (2.3)$$

where the strut coordinate system $\{S\}_i = \{\hat{s}_2, \hat{s}_3, \hat{s}_{23}\}_i$. This frame definition is used for describing the strut motion and to facilitate the derivation of the strut dynamic terms in the equations of motion.

2.2 Equations of Motion

To conduct model-based dynamic parameter identification, one must first formulate the equations of motion used to represent the expected dynamic model of the system. The equations used in this dissertation are based on prior theoretical modeling by references [35, 36, and 39-41]. Assuming a lumped-mass rigid body model, Kane's method is selected for deriving the dynamics equations of motion. The following summarizes the assumptions made.

- Rigid body dynamics of a system of lumped masses and inertias.
- Strut inertias are symmetric about strut longitudinal axis defined along line of action between the strut base and platform joint centers.
- No kinematic joint errors.
- PKM does not have kinematic singularities within its workspace (volume), e.g., $\det[J]$ and $\det[U]$ are not equal to zero for all configurations (these terms are defined later in this chapter).

- Center of mass locations of the extendable portion of the strut (Body E)_i and nonextendable portion of the strut (Body B)_i remain along $(\hat{s}_3)_i$. Both $(m_e)_i$ and $(m_b)_i$ rotate with $(\hat{s}_3)_i$ axis (Refer to Figure 2.2 and 2.3).
- Unmodeled dynamics are attributed to joint imperfections, mass variations, actuator and controller dynamics and the inertial effect of rotational motion of struts about the struts longitudinal axis. These are generally assumed negligible but are retained in the theoretical model to denote sources of error in parameter identification.
- The struts are designed to restrict free spin rotation about its longitudinal axis $(\hat{s}_3)_i$.
- Modified joint friction model $(F_{c_i})_i$ that includes both viscous and Coulomb friction is assumed [36] (model details are provided in Chapter 4). In the analysis presented, the torsional friction within the joints between the base and moving platforms and the struts are assumed negligible (i.e., $(T_{c_i})_i = (T_{v_i})_i = 0, i=1,6$).

2.2.1 Generalized Coordinates

To describe the motion of the 6 degrees of freedom PKM system, a set of 6 generalized coordinates must be chosen. In this system, the generalized coordinates are chosen to be the center of mass location of the moving platform (x_c, y_c, z_c) with reference to the $\{W\}$ coordinate system. For the moving platform's absolute orientation, the x-z-z Euler angles are chosen to avoid computational difficulties and to allow direct integration of the angular velocity. The x-z-z Euler angles represents, sequentially, a rotation ϕ about the z-axis of the $\{W\}$ coordinate system, a rotation θ about the x-axis of the rotated coordinate system, and a rotation ψ about the z-axis of the rotated coordinate system. For this system, to avoid the mathematical singularity, when the platform orientation is in its 'home' position (parallel to the base frame $\{W\}$ and at a specified (x_c, y_c, z_c)), the x-z-z Euler angles are set to $(\phi, \theta, \psi) = (0^\circ, -90^\circ, 0^\circ)$. Thus, at 'home' position, the $\{P\}$ frames are expressed as $\{Y_P = -Z_F, X_P = X_F, Z_P = Y_F\}$.

The symbol $\mathbf{q}$ is used to express the 6x1 vector of the generalized coordinates. For Kane's equations, generalized speeds are also required. The generalized speeds $(\dot{\mathbf{q}})$

correspond to the time derivative of the generalized coordinates. The generalized coordinates and the generalized speeds can be expressed as follows, respectively.

$$\mathbf{q} = [x_c, y_c, z_c, \phi, \theta, \psi]^T \quad (2.4)$$

$$\dot{\mathbf{q}} = [\dot{x}_c, \dot{y}_c, \dot{z}_c, \dot{\phi}, \dot{\theta}, \dot{\psi}]^T \quad (2.5)$$

where $\mathbf{q}$ is the absolute position and orientation coordinates of the platform at its center of mass denoted by point C in Figure 2.6. In equation (2.5), the $(\dot{})$ denotes the time derivative of () . The generalized coordinates can be further expressed as $\mathbf{q} = [{}^w\mathbf{r}_c^T, \boldsymbol{\beta}^T]^T$ where the platform's absolute position (${}^w\mathbf{r}_c$) referenced in $\{W\}$ is

$${}^w\mathbf{r}_c = [x_c \quad y_c \quad z_c]^T \quad (2.6)$$

The orientation coordinates of the moving platform can also be expressed by a vector of the Euler angles, $\boldsymbol{\beta}$, where

$$\boldsymbol{\beta} = [\phi \quad \theta \quad \psi]^T \quad (2.7)$$

Similarly, generalized speed can be expressed as

$$\dot{\mathbf{q}} = [\mathbf{v}_c^T, \dot{\boldsymbol{\beta}}^T]^T \quad (2.8)$$

where $\mathbf{v}_c$ is the absolute linear velocity of the platform's center of mass.

2.2.2 Kinematics

In order to obtain the dynamic response of the PKM system, the position, velocity, and acceleration of all bodies must be expressed in terms of the generalized coordinates. However, to obtain the position, velocity and acceleration of Bodies B_i and E_i in terms of generalized coordinates, the inverse kinematics must be solved. The inverse kinematics is that given position and orientation data of the moving platform, one can determine the

length (L_i) and orientation ($\hat{s}_3$) of the struts. The subscript i in these and the following expressions and equations denote that there is a corresponding expression and set of numerical values for each strut i for $i=1,6$. Without loss of generality, the i subscript will be dropped where it is understood that for all joint, there are $i=1,6$ set of expressions. Given the orientation of the moving platform, a standard 3×3 rotation matrix is defined to map the orientation from the $\{P\}$ coordinate system to $\{F\}$ or $\{W\}$ coordinate systems per the following.

$${}^wR_p = {}^wR_F {}^FR_p = {}^FR_p = \begin{bmatrix} c\psi c\phi - c\theta s\phi s\psi & -s\psi c\phi - c\theta s\phi c\psi & s\theta s\phi \\ c\psi s\phi + c\theta c\phi s\psi & -s\psi s\phi + c\theta c\phi c\psi & -s\theta c\phi \\ s\psi s\theta & c\psi s\theta & c\theta \end{bmatrix} \quad (2.9)$$

where s and c denote the sine and cosine functions, respectively, and ${}^wR_F = [I]$ since $\{F\}$ and $\{W\}$ are parallel frames.

Since the position of the strut joint with the moving platform joint (P_i) is denoted by $(r_{pc})_i$ with reference to frame $\{P\}$, pre-multiplying $(r_{pc})_i$ by ${}^wR_p$ yields the value of P_i position $({}^w r_p)_i$ in the $\{F\}$ coordinate system. Then to complete the mapping to $\{W\}$ frame, the translation from the origin of $\{F\}$ frame to $\{W\}$ frame (${}^w r_c$) must be done. The total mapping of P_i is shown in equation (2.10).

$$({}^w r_p)_i = {}^w r_c + {}^wR_p {}^FR_p (r_{pc})_i \quad (2.10)$$

Once the location of the position P_i is defined, then the strut length and orientation can be determined. Subtracting the location of point b_i from the location of P_i gives the length of the strut.

$$(L\hat{s}_3)_i = ({}^w r_p - {}^w r_b)_i \quad (2.11)$$

Since the unit vector of the strut $(\hat{s}_3)_i$ expressed in $\{W\}$ coordinates is aligned to the individual strut i , it can be found with the following equation.

$$(\hat{s}_3)_i = \frac{({}^W \mathbf{r}_p - {}^W \mathbf{r}_b)_i}{\| {}^W \mathbf{r}_p - {}^W \mathbf{r}_b \|_i} \quad (2.12)$$

The velocity of point P_i $(\mathbf{v}_p)_i$ can be calculated by differentiating $({}^W \mathbf{r}_p)_i$ in equation (2.10) with respect to time.

$$(\mathbf{v}_p)_i = \mathbf{v}_c + (\boldsymbol{\omega}_c \times {}^W \mathbf{R}_p \mathbf{r}_{pc})_i = \mathbf{v}_c + (\boldsymbol{\omega}_c \times {}^W \mathbf{r}_{pc})_i \quad (2.13)$$

where $\boldsymbol{\omega}_c$ is the moving platform's absolute angular velocity defined in the inertial frame $\{W\}$ coordinates.

Since the velocity of point P must be expressed in terms of the generalized coordinates, an expression for $\boldsymbol{\omega}_c$ in terms of the generalized coordinates and generalized speeds must be found. That is,

$$\boldsymbol{\omega}_c = \begin{bmatrix} 0 & c\phi & s\phi s\theta \\ 0 & s\phi & -c\phi s\theta \\ 1 & 0 & c\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = [\mathbf{U}] \dot{\mathbf{\beta}} \quad (2.14)$$

where $[\mathbf{U}]$ is the matrix transformation from the Euler angular rates to the absolute angular velocity vector for the platform, $\boldsymbol{\omega}_c$. The derivation of matrix $[\mathbf{U}]$ is provided in Appendix A.

When substituting equation (2.14) into (2.13), the equation yields the velocity of point P_i in the $\{W\}$ coordinate system as a function of the generalized coordinates and generalized speeds. The component of this velocity in the $(\hat{s}_3)_i$ direction is the strut i extension rate, which can be obtained by dotting the velocity of point P_i with

the $(\hat{\mathbf{s}}_i)_i$ vector as shown in equation (2.15). This operation results in the magnitude and sense of the strut extension rate.

$$\dot{L}_i = (\mathbf{v}_p \cdot \hat{\mathbf{s}}_i)_i \quad (2.15)$$

In addition to the equation (2.10), the location of point P_i can also be found as

$$(\mathbf{r}_p)_i = (L\hat{\mathbf{s}}_i + \mathbf{r}_b)_i \quad (2.16)$$

Differentiating equation (2.16) yields the velocity of point P_i.

$$(\mathbf{v}_p)_i = (\dot{L}\hat{\mathbf{s}}_i + \tilde{\boldsymbol{\omega}}_i \times L\hat{\mathbf{s}}_i)_i \quad (2.17)$$

where the total strut angular velocity in {W} coordinates is specified as

$$(\tilde{\boldsymbol{\omega}}_i)_i = (\omega_{s_{i2}})_i (\hat{\mathbf{s}}_2)_i + (\omega_{s_{i1}})_i (\hat{\mathbf{s}}_1)_i + (\omega_{s_{i3}})_i (\hat{\mathbf{s}}_3)_i \quad (2.18)$$

Since $(\hat{\mathbf{s}}_i \times \hat{\mathbf{s}}_i)_i = 0$, $(\mathbf{v}_p)_i$ can also be written as

$$(\mathbf{v}_p)_i = \dot{L}\hat{\mathbf{s}}_i + \boldsymbol{\omega}_i \times L\hat{\mathbf{s}}_i \quad (2.19)$$

and the strut angular velocity component $(\boldsymbol{\omega}_i)_i$ without the angular velocity about $\hat{\mathbf{s}}_i$ axis is

$$(\boldsymbol{\omega}_i)_i = (\omega_{s_{i2}}\hat{\mathbf{s}}_2 + \omega_{s_{i3}}\hat{\mathbf{s}}_3)_i \quad (2.20)$$

which can also be expressed as

$$(\boldsymbol{\omega}_i)_i = \left(\left(\frac{1}{L} \right) (\hat{\mathbf{s}}_i \times \mathbf{v}_p)_i \right) \quad (2.21)$$

It should be noted

$$\boldsymbol{\omega}_i \cdot \hat{\mathbf{s}}_i = 0$$

Hence, a physical setup, the struts are typically designed to restrict free spin rotation about its longitudinal axis $(\hat{\mathbf{s}}_i)_i$, therefore, the dot product of $(\tilde{\boldsymbol{\omega}}_i)_i$ and $(\hat{\mathbf{s}}_i)_i$ are zero

$((\tilde{\omega}_s \cdot \hat{s}_3)_i = 0)$. Thus, the strut angular velocity about $(\hat{s}_3)_i$ axis is $(\omega_s)_i$ and the strut entities Body B_i and Body E_i have the same angular velocities $(\omega_E)_i = (\omega_B)_i = (\tilde{\omega}_s)_i$. In actuality, depending on joint designs for joint b_i and P_i there is small rotations about $(\hat{s}_3)_i$, but the dynamics due to $\tilde{\omega}_s$ in the $\hat{s}_3$ are considered negligible. Hence, from this point forward, ω_E and ω_B are assumed equal to ω_s .

The notation for the absolute locations and velocities of center of mass of Bodies E_i and B_i and the platform joint P_i can be expressed as:

$$({}^u \mathbf{r}_E)_i = (L_E \hat{s}_3 + {}^u \mathbf{r}_b)_i \quad (2.22)$$

$$({}^u \mathbf{r}_B)_i = (L_B \hat{s}_3 + {}^u \mathbf{r}_b)_i \quad (2.23)$$

$$(\mathbf{v}_E)_i = (\dot{L}_E \hat{s}_3 + \omega_E \times (L_E \hat{s}_3))_i \quad (2.24)$$

$$(\mathbf{v}_B)_i = (\omega_B \times (L_B \hat{s}_3))_i \quad (2.25)$$

$$(\mathbf{v}_P)_i = \mathbf{v}_c + (\omega_c \times {}^u \mathbf{r}_{pc})_i = (\dot{L} \hat{s}_3 + \omega_s \times (L \hat{s}_3))_i \quad (2.26)$$

Strut angular accelerations are determined from the linear velocities and accelerations. To derive the equations of motion in terms of only the platform's motion, the velocities and accelerations of the length of the strut variable (L) can be defined as follows:

From equation (2.15),

$$\dot{L}_i = \dot{L}_{E_i} = (\mathbf{v}_P \cdot \hat{s}_3)_i \quad (2.27)$$

$$\begin{aligned} \ddot{L}_i = \ddot{L}_{E_i} &= \frac{d}{dt} (\mathbf{v}_P \cdot \hat{s}_3)_i = \frac{d}{dt} (\mathbf{v}_P)_i \cdot (\hat{s}_3)_i + (\mathbf{v}_P)_i \cdot \frac{d}{dt} (\hat{s}_3)_i \\ &= \frac{d}{dt} (\mathbf{v}_c + (\omega_c \times {}^u \mathbf{r}_{pc}))_i \cdot (\hat{s}_3)_i + (\mathbf{v}_P)_i \cdot (\omega_s \times \hat{s}_3)_i \end{aligned} \quad (2.28)$$

$$\begin{aligned} \mathbf{a} &= \mathbf{a} + (\mathbf{u} \times \mathbf{r}^s) \times (\boldsymbol{\omega}^s + \boldsymbol{\omega} \times \mathbf{s}_s) \times (\mathbf{r}^s \times \mathbf{s}_s) \\ &= \mathbf{a} + (\mathbf{u} \times \mathbf{r}^s) \times (\boldsymbol{\omega}^s + \boldsymbol{\omega} \times \mathbf{s}_s) \times (\mathbf{r}^s \times \mathbf{s}_s) + L_s \mathbf{s}_s \times (\boldsymbol{\omega}^s + \boldsymbol{\omega} \times \mathbf{s}_s) \cdot (\boldsymbol{\omega}^s + \boldsymbol{\omega} \times \mathbf{s}_s) \end{aligned}$$

$$(2.29) \quad \boldsymbol{\omega} \times \mathbf{s}_s = \left[\boldsymbol{\omega} \times \mathbf{s}_s + \boldsymbol{\omega} + \boldsymbol{\omega} \times \mathbf{s}_s \right] \times \mathbf{s}_s = (\boldsymbol{\omega} \times \mathbf{s}_s + \boldsymbol{\omega} + \boldsymbol{\omega} \times \mathbf{s}_s) \times \mathbf{s}_s = -\mathbf{s}_s$$

$$(2.30) \quad \boldsymbol{\omega} \times \mathbf{s}_s \cdot (\boldsymbol{\omega} \times \mathbf{s}_s) = (\boldsymbol{\omega} \times \mathbf{s}_s + \boldsymbol{\omega} + \boldsymbol{\omega} \times \mathbf{s}_s) \cdot \mathbf{s}_s = \boldsymbol{\omega} \cdot \mathbf{s}_s$$

$$\text{where } \|\boldsymbol{\omega}\| = \sqrt{\boldsymbol{\omega} \cdot \boldsymbol{\omega}}$$

$$(2.31) \quad \ddot{L}_s = \mathbf{a} \cdot \mathbf{r}^s = (\mathbf{a} + (\mathbf{u} \times \mathbf{r}^s) \times (\boldsymbol{\omega}^s + \boldsymbol{\omega} \times \mathbf{s}_s) \times (\mathbf{r}^s \times \mathbf{s}_s) + \boldsymbol{\omega} \times \mathbf{s}_s) \cdot \mathbf{r}^s$$

The linear acceleration of the center of mass of Bodies E and B and the moving

platform can be determined with simply differentiating equations (2.24) to (2.26). i.e.,

$$(2.32) \quad \begin{aligned} \mathbf{a} &= \mathbf{a} + (\mathbf{u} \times \mathbf{r}^s) \times (\boldsymbol{\omega}^s + \boldsymbol{\omega} \times \mathbf{s}_s) \times (\mathbf{r}^s \times \mathbf{s}_s) \\ &= \frac{dp}{dt} \left(L_s \mathbf{s}_s \times \boldsymbol{\omega} + \boldsymbol{\omega} \times \mathbf{s}_s \right) \\ &= (\boldsymbol{\omega}^s + \boldsymbol{\omega} \times \mathbf{s}_s) \times (\mathbf{r}^s \times \mathbf{s}_s) + 2L_s \boldsymbol{\omega} \times \mathbf{s}_s + \boldsymbol{\omega} \times \mathbf{s}_s \times (\boldsymbol{\omega} \times \mathbf{s}_s) \end{aligned}$$

$$(2.33) \quad \begin{aligned} \mathbf{a} &= \frac{dp}{dt} \left(L_s \mathbf{s}_s \times \boldsymbol{\omega} + \boldsymbol{\omega} \times \mathbf{s}_s \right) \\ &= L_s \ddot{L}_s \mathbf{s}_s + 2L_s \boldsymbol{\omega} \times \mathbf{s}_s + \boldsymbol{\omega} \times \mathbf{s}_s \times L_s \mathbf{s}_s + \boldsymbol{\omega} \times \mathbf{s}_s \times (\boldsymbol{\omega} \times \mathbf{s}_s) \end{aligned}$$

$$(2.34) \quad \mathbf{a} = (\mathbf{u} \times \mathbf{r}^s) \times L_s \mathbf{s}_s + \boldsymbol{\omega} \times \mathbf{s}_s \times L_s \mathbf{s}_s$$

Since for each strut the corresponding angular strut velocities of the Bodies E and B are the same, the angular accelerations for these bodies are the same and can be derived by

crossing $\mathbf{a}$ with $\mathbf{s}_s$, i.e.,

$$(2.35) \quad \mathbf{s}_s \times \mathbf{a} = \mathbf{s}_s \times (\mathbf{s}_s \times L_s \mathbf{s}_s + 2L_s \boldsymbol{\omega} \times \mathbf{s}_s + \boldsymbol{\omega} \times \mathbf{s}_s \times L_s \mathbf{s}_s) = L_s \mathbf{s}_s \times \mathbf{s}_s$$

$$(2.36) \quad \therefore \mathbf{a} \cdot \mathbf{s}_s = L_s \mathbf{s}_s \cdot \mathbf{s}_s = L_s$$

The angular acceleration of the moving platform can be obtained by differentiating ω_c given in equation (2.14).

$$\alpha_c = [\mathbf{U}]\ddot{\mathbf{p}} + [\dot{\mathbf{U}}]\dot{\mathbf{p}} \quad (2.37)$$

$$\text{where } [\dot{\mathbf{U}}] = \begin{bmatrix} 0 & -s\phi\dot{\phi} & (c\phi s\theta\dot{\phi} + s\phi c\theta\dot{\theta}) \\ 0 & c\phi\dot{\phi} & (s\phi s\theta\dot{\phi} - c\phi c\theta\dot{\theta}) \\ 0 & 0 & -s\theta\dot{\theta} \end{bmatrix} \quad (2.38)$$

2.2.3 Kane's Equation

Using the generalized coordinates defined in equation (2.1), Kane's method is now used to derive the equations of motion. Its generic form is:

$$\sum_{j=1}^w \left(\frac{\partial \mathbf{v}_j}{\partial \dot{q}_k} \cdot \{\mathbf{F}_j - \mathbf{F}_j^*\} \right) + \sum_{j=1}^w \left(\frac{\partial \boldsymbol{\omega}_j}{\partial \dot{q}_k} \cdot \{\mathbf{T}_j - \mathbf{T}_j^*\} \right) = 0 \quad k=1,2,\dots,n \quad (2.39)$$

where the $\frac{\partial \mathbf{v}_j}{\partial \dot{q}_k}$ and $\frac{\partial \boldsymbol{\omega}_j}{\partial \dot{q}_k}$ are the partial derivatives of the body j's absolute linear and angular velocities with respect to the n^{th} generalized speed $\dot{q}_n$, respectively. Here w is the number of rigid bodies and n is the number of the generalized coordinates. The $\mathbf{F}_j$ is the resultant external force acting on each body j, while $\mathbf{F}_j^*$ is the inertial force of body j. Similarly, $\mathbf{T}_j$ and $\mathbf{T}_j^*$ represent the resultant external torque/moments and inertial moment of body j about its center of mass, respectively. Due to equation (2.39) being summations of vector dot products and there are n generalized coordinates, the Kane's method results in a set of n scalar equations.

For the Stewart-Gough PKM, there exists one moving platform (plat), six-extendable strut components (Body E), and six-base strut components (Body B). Thus, rewriting Kane's equation for the Stewart-Gough PKM:

$$\begin{aligned}
& \sum_{i=1}^6 \left[\frac{\partial \mathbf{v}_E}{\partial \dot{q}_k} \cdot \{\mathbf{F}_E - \mathbf{F}_E^*\} \right]_i + \sum_{i=1}^6 \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \dot{q}_k} \cdot \{\mathbf{T}_E - \mathbf{T}_E^*\} \right]_i \\
& + \sum_{i=1}^6 \left[\frac{\partial \mathbf{v}_B}{\partial \dot{q}_k} \cdot \{\mathbf{F}_B - \mathbf{F}_B^*\} \right]_i + \sum_{i=1}^6 \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \dot{q}_k} \cdot \{\mathbf{T}_B - \mathbf{T}_B^*\} \right]_i \\
& + \frac{\partial \mathbf{v}_c}{\partial \dot{q}_k} \cdot \{\mathbf{F}_c - \mathbf{F}_c^*\} + \frac{\partial \boldsymbol{\omega}_c}{\partial \dot{q}_k} \cdot \{\mathbf{T}_c - \mathbf{T}_c^*\} = 0
\end{aligned} \tag{2.40}$$

$k=1,2,\dots,6$

where $(\mathbf{v}_E, \boldsymbol{\omega}_E)$ and $(\mathbf{v}_B, \boldsymbol{\omega}_B)$ are the absolute (linear velocity, angular velocity) of Body E and Body B, respectively.

The resultant external forces for each body type are listed below.

$$\mathbf{F}_c = m_p \mathbf{g} + \mathbf{F}_{tool} \tag{2.41}$$

$$\mathbf{F}_E = m_e \mathbf{g} + (F_a - F_c - F_{m/b}) \hat{\mathbf{s}}_3 \tag{2.42}$$

$$\mathbf{F}_B = (F_c + F_{m/b} - F_a) \hat{\mathbf{s}}_3 + m_b \mathbf{g} \tag{2.43}$$

where m_p is the mass of the platform and $\mathbf{g}$ is the gravitational field vector. $\mathbf{F}_{tool}$ is the applied tool and/or payload force, such as the reaction forces generated by deburring or other cutting operations. The actuator force, F_a , acts along the vector $\hat{\mathbf{s}}_3$ with a magnitude and sense of F_a , however, the friction force ($F_c \hat{\mathbf{s}}_3$) and the ball inertial screw forces ($F_{m/b} \hat{\mathbf{s}}_3$) act in the opposite direction. The $F_{m/b}$ are part of the original unmodeled set of dynamic terms but are included here for enlightenment. For the external force acting on Body B ($\mathbf{F}_B$), the forces, ($F_c \hat{\mathbf{s}}_3$, $F_{m/b} \hat{\mathbf{s}}_3$, and $F_a \hat{\mathbf{s}}_3$), have the same magnitude but in equal and opposite directions when compared to those acting on Body E ($\mathbf{F}_E$). For all bodies, gravity forces are defined by $m \mathbf{g}$ terms. In the simulation so far, friction and

ball screw forces are assumed to be zero. Since the friction forces in the spherical and Hooke joint are assumed to be zero and the motor torques are modeled as equivalent linear actuator forces, there are no other applied external torque/moments to the system. I.e.,

$$\mathbf{T}_c = \mathbf{T}_E = \mathbf{T}_B = \mathbf{0} \quad (2.44)$$

For the inertial terms ($\mathbf{F}_j^*$, $\mathbf{T}_j^*$), it can be written as:

$$\mathbf{F}_j^* = m_j \mathbf{a}_j \quad (2.45)$$

$$\mathbf{T}_j^* = \mathbf{a}_j \circ \mathbf{I}_j'' + \boldsymbol{\omega}_j \times (\mathbf{I}_j'' \circ \boldsymbol{\omega}_j) \quad (2.46)$$

where $\mathbf{a}_j$ is the absolute linear acceleration of the center of mass of body j ; $\boldsymbol{\omega}_j$ and $\mathbf{a}_j$ are the absolute angular velocity and acceleration of body j , respectively; $\mathbf{I}_j''$ is the dyadic term for the inertia of the j^{th} body about its center of mass; ‘ $\circ$ ’ denotes dyadic product of the dyadic term with a vector; and ‘ $\times$ ’ denotes the cross product of vectors. The reader is referred to reference [40] for detailed description of dyadics.

The partial derivative terms from Kane’s equation for the Stewart-Gough PKMs are defined next. Using vector-matrix notation and partition matrices, these derivatives can be written as follows.

$$\frac{\partial \mathbf{v}_e}{\partial \mathbf{v}_e} = [\mathbf{I}], \quad \frac{\partial \mathbf{v}_e}{\partial \dot{\boldsymbol{\beta}}} = [\mathbf{0}], \quad \frac{\partial \boldsymbol{\omega}_e}{\partial \mathbf{v}_e} = [\mathbf{0}], \quad \frac{\partial \boldsymbol{\omega}_e}{\partial \dot{\boldsymbol{\beta}}} = [\mathbf{U}] \quad (2.47)$$

where $[\mathbf{I}]$ is a (3x3) identity matrix and $[\mathbf{0}]$ is a (3x3) zero matrix. For each strut i , the following equations are stated with subscript i omitted for brevity. Note, $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors of $\{W\}$, X_w , Y_w , Z_w respectively. The $\dot{}$ in $\begin{bmatrix} \dot{} \\ \vdots \end{bmatrix}$ are used in forming partitions in the $\begin{bmatrix} \\ \vdots \end{bmatrix}$ matrix.

$$\frac{\partial \mathbf{v}_E}{\partial \mathbf{v}_e} = (\hat{\mathbf{s}}_3 \hat{\mathbf{s}}_3^T) + \left(\frac{L_E}{L} \right) \left[(\hat{\mathbf{s}}_3 \times \hat{i}) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times \hat{j}) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times \hat{k}) \times \hat{\mathbf{s}}_3 \right] \quad (2.48)$$

$$\frac{\partial \mathbf{v}_B}{\partial \mathbf{v}_e} = \left(\frac{L_B}{L} \right) \left[(\hat{\mathbf{s}}_3 \times \hat{i}) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times \hat{j}) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times \hat{k}) \times \hat{\mathbf{s}}_3 \right] \quad (2.49)$$

$$\frac{\partial \boldsymbol{\omega}_S}{\partial \mathbf{v}_e} = \frac{\partial \boldsymbol{\omega}_B}{\partial \mathbf{v}_e} = \frac{\partial \boldsymbol{\omega}_E}{\partial \mathbf{v}_e} = \left(\frac{1}{L} \right) \left[(\hat{\mathbf{s}}_3 \times \hat{i}) \quad : \quad (\hat{\mathbf{s}}_3 \times \hat{j}) \quad : \quad (\hat{\mathbf{s}}_3 \times \hat{k}) \right] \quad (2.50)$$

$$\begin{aligned} \frac{\partial \mathbf{v}_E}{\partial \boldsymbol{\beta}} &= \hat{\mathbf{s}}_3 \left[{}^w \mathbf{r}_{pc} \times \hat{\mathbf{s}}_3 \right]^T [\mathbf{U}] \\ &+ \left(\frac{L_E}{L} \right) \left[(\hat{\mathbf{s}}_3 \times (\llbracket \mathbf{U} \rrbracket_{Col 1} \times {}^w \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times (\llbracket \mathbf{U} \rrbracket_{Col 2} \times {}^w \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times (\llbracket \mathbf{U} \rrbracket_{Col 3} \times {}^w \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \right] \end{aligned} \quad (2.51)$$

$$\frac{\partial \mathbf{v}_B}{\partial \boldsymbol{\beta}} = \left(\frac{L_B}{L} \right) \left[(\hat{\mathbf{s}}_3 \times (\llbracket \mathbf{U} \rrbracket_{Col 1} \times {}^w \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times (\llbracket \mathbf{U} \rrbracket_{Col 2} \times {}^w \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times (\llbracket \mathbf{U} \rrbracket_{Col 3} \times {}^w \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \right] \quad (2.52)$$

$$\frac{\partial \boldsymbol{\omega}_S}{\partial \boldsymbol{\beta}} = \frac{\partial \boldsymbol{\omega}_B}{\partial \boldsymbol{\beta}} = \frac{\partial \boldsymbol{\omega}_E}{\partial \boldsymbol{\beta}} = \left(\frac{1}{L} \right) \left[(\hat{\mathbf{s}}_3 \times (\llbracket \mathbf{U} \rrbracket_{Col 1} \times {}^w \mathbf{r}_{pc})) \quad : \quad (\hat{\mathbf{s}}_3 \times (\llbracket \mathbf{U} \rrbracket_{Col 2} \times {}^w \mathbf{r}_{pc})) \quad : \quad (\hat{\mathbf{s}}_3 \times (\llbracket \mathbf{U} \rrbracket_{Col 3} \times {}^w \mathbf{r}_{pc})) \right] \quad (2.53)$$

where $[\mathbf{U}]_{Col i}$ is the i^{th} column of matrix $[\mathbf{U}]$.

2.2.4 Equations of Motion (Force-Based Model)

Once the above equations are substituted into the Kane's equation, a set of six scalar equations will result, where the six $(\mathbf{F}_a)_i$ terms can be factored out. The force-based model uses the resulting model expression for the vector of these six actuator force terms $(\mathbf{F}_a)_i$. When expressing these six equations of motion in an ordered, compact vector-matrix form per the defined generalized coordinates, the coefficients of these $(\mathbf{F}_a)_i$ terms yield a pseudo-Jacobian matrix

$$\mathbf{J} = \begin{bmatrix} (\hat{\mathbf{s}}_3)_1 & \vdots & \cdots & \vdots & (\hat{\mathbf{s}}_3)_6 \\ [\mathbf{U}]^T (\mathbf{r}_{pc} \times \hat{\mathbf{s}}_3)_1 & \vdots & \cdots & \vdots & [\mathbf{U}]^T (\mathbf{r}_{pc} \times \hat{\mathbf{s}}_3)_6 \end{bmatrix} \quad (2.54)$$

analogous to the traditional Jacobian definition for PKMs, refer to Appendix B (equation (B.33)-(B.36)) for further derivation details. That is, the Pseudo-Jacobian matrix can be physically interpreted as the pose of machine expressed in the Euler angles, especially if one takes into account that the $[\mathbf{U}]$ matrix, defined in equation (2.14), is the transformation matrix from the Euler angular rates to the absolute angular velocity of the platform in $\{\mathbf{W}\}$ coordinates. Solving for the actuator forces yields.

$$\begin{aligned} \mathbf{F}_a = \mathbf{J}^{-1} & \left[\begin{bmatrix} \mathbf{F}_{pm} \\ \mathbf{T}_{pm} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{sm} \\ \mathbf{T}_{sm} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{cor} \\ \mathbf{T}_{cor} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{cen} \\ \mathbf{T}_{cen} \end{bmatrix} \right. \\ & \left. + \begin{bmatrix} \mathbf{F}_{grp} \\ \mathbf{T}_{grp} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{grs} \\ \mathbf{T}_{grs} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{frict} \\ \mathbf{T}_{frict} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{ext} \\ \mathbf{T}_{ext} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{m/b} \\ \mathbf{T}_{m/b} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{unbd} \\ \mathbf{T}_{unbd} \end{bmatrix} \right] \end{aligned} \quad (2.55)$$

where $\mathbf{F}_a$ denotes a vector of the magnitude and sense of the six actuator forces, each applied along their respective strut axes $(\hat{\mathbf{s}}_3)_i$. In the above equation of motion, equation (2.55), the dynamic forces and moment contributions and the external loads are grouped by their source. Details of the components of these forces and moments are expressed in Appendix B. For the platform acceleration dependent terms, the forces and moments are grouped as $\begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix}^T$. The gravitational dependent terms for the platform are grouped in $\begin{bmatrix} \mathbf{F}_{grp}^T & \mathbf{T}_{grp}^T \end{bmatrix}^T$. Similarly, strut acceleration and the gravitation dependent terms in equation (2.55) are grouped in $\begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix}^T$ and $\begin{bmatrix} \mathbf{F}_{grs}^T & \mathbf{T}_{grs}^T \end{bmatrix}^T$, respectively. The Coriolis and centripetal acceleration-like terms are grouped in

$\begin{bmatrix} \mathbf{F}_{cor}^T & \mathbf{T}_{cor}^T \end{bmatrix}^T$ and $\begin{bmatrix} \mathbf{F}_{cen}^T & \mathbf{T}_{cen}^T \end{bmatrix}^T$, respectively, where they contain terms that are a

function of strut and platform velocities products. As expected, the resulting equations of motion are highly nonlinear. Other sources of nonlinearities are joint frictions which are denoted by the $\begin{bmatrix} \mathbf{F}_{frict}^T & \mathbf{T}_{frict}^T \end{bmatrix}^T$ and are dependent on the PKM instantaneous configuration, velocities and friction modeled. The $\begin{bmatrix} \mathbf{F}_{frict}^T & \mathbf{T}_{frict}^T \end{bmatrix}^T$ terms are set to zero for the test without friction, where the friction modeling (Chapter 4) is added for the test with friction. The $\begin{bmatrix} \mathbf{F}_{ext}^T & \mathbf{T}_{ext}^T \end{bmatrix}^T$ consists of the external loads acting on the PKM. The $\begin{bmatrix} \mathbf{F}_{m/h}^T & \mathbf{T}_{m/h}^T \end{bmatrix}^T$ represents the terms due to the motor and ball screw dynamics. The $\begin{bmatrix} \mathbf{F}_{umdl}^T & \mathbf{T}_{umdl}^T \end{bmatrix}^T$ denotes the collection of the other unmodeled dynamics term. In the remainder of this dissertation, the $\begin{bmatrix} \mathbf{F}_{m/h}^T & \mathbf{T}_{m/h}^T \end{bmatrix}^T$ and $\begin{bmatrix} \mathbf{F}_{umdl}^T & \mathbf{T}_{umdl}^T \end{bmatrix}^T$ terms are assumed to be negligible. Table 2.1 shows a summary of the components associated with each term of equation (2.55). For further details on generating the equations of motion, the reader is referred to Walker [39] and Register [36].

2.2.5 The Energy Method

The prior sections in this dissertation were dedicated to obtaining the full set of dynamic equations of motion to formulate the model basis. For the parameter identification algorithms, the implementation of the identification algorithm with the dynamic equations of motion as the model basis involves measuring position, velocity, acceleration and actuator forces as the inputs to the identification process. Measurements of velocity and acceleration are typically prone to errors. When data is not directly measurable but must be calculated numerically from other measured data, e.g., position data, errors can rapidly grow in magnitude especially with each level of differentiation

that is calculated using data containing noise. To circumvent this problem, one can instrument the setup with better quality sensors, which tends to drive up costs and require greater engineering expertise. An alternative would be to use as the model base the energy method suggested by Bhattacharya [33]. In doing so, one does not need to measure accelerations, thus eliminating one level of noise amplification in calculated inputs to the identification algorithm. Thus, an energy-based model approach is investigated as a potential alternative to the equation of motion based approach, referred to as the force-based approach in this dissertation.

For the energy-based method, the PKM is assumed to move in a singular-free trajectory, permitting an energy difference determination for time step $(x_k, \dot{x}_k)$ through $(x_{k+1}, \dot{x}_{k+1})$. The energy equations developed by Bhattacharya [33] are defined as the following scalar function.

$$h_{k+1}(x_{k+1}, \dot{x}_{k+1}, w_{k+1}) - h_k(x_k, \dot{x}_k, w_k) = \int F \cdot \dot{x} dt = \sum_{i=1}^n (F_{a_i})_k (\Delta L_i)_k \quad (2.56)$$

where $h_k(x_k, \dot{x}_k, w_k)$ is the total energy (kinetic plus potential energy) at the k^{th} states, $h_{k+1}(x_{k+1}, \dot{x}_{k+1}, w_{k+1})$ is total energy (kinetic plus potential energy) at the $(k+1)^{\text{th}}$ state, w_k and w_{k+1} are the current k^{th} and $(k+1)^{\text{th}}$ inertia parameter estimates. $(F_{a_i})_k$ is the i^{th} strut force at the k^{th} state, $(\Delta L_i)_k$ is the change in the i^{th} strut length at k^{th} state, n is the number of struts.

For the PKMs modeled in this dissertation, the mass matrix is the same as that defined in Appendix B (equations (B.2) - (B.8) and (B.25)). The energy at k^{th} state is expressed as follows.

$$h_k(x_k, \dot{x}_k, w_k) = T_k + V_k \quad (2.57)$$

where the kinetic energy, $T_k = \frac{1}{2} [\mathbf{v}_c^T, \dot{\mathbf{p}}^T] [\mathbf{M}] \begin{bmatrix} \mathbf{v}_c \\ \dot{\mathbf{p}} \end{bmatrix}$ is a scalar quantity on dependent of the

choice of generalized coordinates as illustrated in Appendix B, equations (B.26)-(B.32)

and the total potential energy in the system is given by

$$V_k = (-1) \left[m_p \mathbf{g}^T \mathbf{r}_c + \sum_{i=1}^6 [m_i \mathbf{g}^T \mathbf{r}_E] + \sum_{i=1}^6 [m_b \mathbf{g}^T \mathbf{r}_B] \right].$$

2.3 Parameter Identification Algorithms

In this section, the proposed methodology for identifying the dynamic system parameters will be shown. In general, the minimal set of dynamic parameters consists of ten parameters per rigid body: the mass, inertia tensor and the locations of the center of mass per rigid body in the system. These terms can be extracted from the system's theoretical model-base equations forming the set of potentially identifiable parameters. As the system equations become more complex and nonlinear, the extraction of the parameters in forming the traditional vector set of identifiable parameters, as required by least squares (LS) and extended Kalman filter (EKF) methods, becomes insurmountable. Since the ultimate goal for this dissertation is to develop a methodology for determining accurate dynamic system parameter values for parallel kinematic machines (a highly nonlinear system), the ease of implementing the estimation process is critical. The system identification process for parameter estimation will be chosen between the following methods:

- Least square (LS)
- Extended Kalman filter (EKF)
- Unscented Kalman filter (UKF)

2.3.1 Least Squares Method (LS)

Since the parameter estimation algorithm has a long history, the pros and cons of the LS method are well known to the readers. For linear systems, many researchers [23,32] have used the LS method for system identification. The system identification process with a linear trajectory and linear parametric modeling is successful for most cases. However, when nonlinear terms are present, the nonlinearity of the system may cause defects in the identification analysis and lead to convergence problems.

The LS method is based on the equation (on k^{th} iteration step)

$$\text{Min } f = \sum_{i=1}^m e_i^2 \quad (2.58)$$

where reference residual, $e_i = (y_k - d_k)_i$, the y_k is the updated output (calculated from measured states (x_k) and estimated parameters (w_k)) for k^{th} iteration step, the d_k is the measured (desired) output, and m is the number of residual equations.

The linearized form yields

$$e_i = e_i|_{w_{ko}} + \frac{\partial e_i}{\partial w_j} \Big|_{w_{ko}} \cdot \Delta w_j \quad i=1,m \quad (2.59)$$

where w_{ko} is the nominal value for parameters at k^{th} iteration step, $e_i|_{w_{ko}}$ is the i^{th} residual evaluated at w_{ko} , $\frac{\partial e_i}{\partial w_j} \Big|_{w_{ko}}$ is the partial derivative of the j^{th} parameter and is evaluated at w_{ko} , the Δw_j is the change in the n^{th} parameter for $j=1,n$, and n is the number of parameters. For m equations or poses, the vector equation form is

$$\mathbf{e}_k = \mathbf{e}_k|_{w_{ko}} + J_{LS,k} \Delta w_k + H.O.T. \quad (2.60)$$

where $\mathbf{e}_k = [e_1 \ e_2 \ \cdots \ e_m]^T$, $\mathbf{e}_k|_{w_{ko}} = [e_1|_{w_{ko}} \ e_2|_{w_{ko}} \ \cdots \ e_m|_{w_{ko}}]$, $\Delta w = [\Delta w_1 \ \Delta w_2 \ \cdots \ \Delta w_n]^T$

$$J_{LS,k} = \begin{bmatrix} \frac{\partial e_1}{\partial w_1} & \dots & \frac{\partial e_1}{\partial w_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial e_m}{\partial w_1} & \dots & \frac{\partial e_m}{\partial w_n} \end{bmatrix}_{w_{k0}}, \quad (2.61)$$

and *H.O.T.* stands for higher order terms. If ignoring the *H.O.T.*, the objective function is

$$\text{Min } f = (\mathbf{e}_k |_{w_{k0}} + J_{LS,k} \Delta w_k)^T (\mathbf{e}_k |_{w_{k0}} + J_{LS,k} \Delta w_k) \quad (2.62)$$

The minimum value is derived by setting $\frac{\partial f}{\partial \Delta w_k} = 0$ together with $\frac{\partial^2 f}{\partial \Delta w_k^2} > 0$.

$$\frac{\partial f}{\partial \Delta w_k} = -2J_{LS,k}^T \mathbf{e}_k |_{w_{k0}} + (J_{LS,k}^T J_{LS,k} + J_{LS,k}^T J_{LS,k}) \Delta w_k = 0 \quad (2.63)$$

which yields

$$\Delta w_k = (J_{LS,k}^T J_{LS,k})^{-1} J_{LS,k}^T \mathbf{e}_k |_{w_{k0}} \quad (2.64)$$

and

$$w_{k+1} = w_k + \Delta w_k \quad (2.65)$$

The LS program iterates until the largest value in Δw_k is less than desired tolerance to obtain the estimation of w_k .

2.3.2 Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF)

The EKF method has widely been used because of its benefits [36]. For the PKM system, the EKF's ability to handle noisy data and highly non-linear, multi-input/multi-output systems is attractive. However recently, Wan and Van der Merwe [38] demonstrated the UKF as an alternative to EKF, pointing out the flaws and shortcomings of the EKF. The EKF involves a linearization approximation within its algorithm when

applied to nonlinear systems. This causes an elimination of the expectations for the optimal prediction terms, which induces large errors in the true posterior mean and covariance. Thus, Wan and Van der Merwe [38] proposed the UKF method to overcome these algorithm limitations. Instead of using a first-order linearization approximation, the UKF uses an optimal prediction of the input. The technique that eliminates the first-order approximations is called Unscented Transformation (UT). The UKF estimates in two steps. The first step evaluates and estimates the existing value of the initial guess of the parameters using the unscented transformation. In the second step, the parameterized vector is estimated using a Kalman gain (weighting) technique. The following section details the theory of the EKF and UKF and further highlights the main differences between EKF and the UKF.

2.3.2.1 Extended Kalman Filter Parameter Estimation

For the basic state-space estimation, recursive estimation for input x_k can be expressed as

$$\hat{x}_k = (\text{prediction of } x_k) + k_k \cdot [y_k - (\text{prediction of } y_k)] \quad (2.66)$$

where k_k is the optimal gain term. The optimal terms for this recursion are

$$\begin{aligned} \hat{x}_{\bar{k}} &= E[F(\hat{x}_{k-1}, v_{k-1})] \\ k_k &= P_{x_k y_k} P_{x_k y_k}^{-1} \\ \hat{y}_{\bar{k}} &= E[H(\hat{x}_{\bar{k}}, n_k)] \end{aligned} \quad (2.67)$$

where the optimal prediction of x_k is expressed as $\hat{x}_{\bar{k}}$, and the optimal prediction of the y_k is expressed as $\hat{y}_{\bar{k}}$. The v_{k-1} represents the process noise, n_k stands for the measurement noise. The $E[F(\cdot)]$ denotes the expected value of the function $F(\cdot)$ and the $E[H(\cdot)]$ denotes the expected value of the function $H(\cdot)$. The determination of this

expectation function depends on the estimation procedure selected. The $\hat{P}_{x_i, y_i}$ function is defined as the covariance of the $\hat{x}_{\bar{k}}$ and $\hat{y}_{\bar{k}}$ functions. The Kalman filter and extended Kalman filter techniques approximate these quantities using linearization, i.e.,

$$\begin{aligned}\hat{x}_{\bar{k}} &\approx F(\hat{x}_{k-1}, \bar{v}) \\ k_k &\approx \hat{P}_{x_i, y_i} P_{x_i, y_i}^{-1} \\ \hat{y}_{\bar{k}} &\approx H(\hat{x}_{\bar{k}}, \bar{n})\end{aligned}\quad (2.68)$$

These approximations, however, can lead to large errors [38] when applied to nonlinear systems.

2.3.2.2 UKF Parameter Estimation

The UKF finds its roots in the 1995 development of the Unscented Transformation (UT) used for filtering nonlinear systems [42, 43]. Julier and Uhlmann [44] later applied the UT to extend Kalman filtering in the development of a generalized method for estimation of nonlinear systems. The classic nonlinear mapping takes the form.

$$y_k = G(x_k, w_k) \quad (2.69)$$

where x_k is the measured states and y_k is the updated output (calculated from measured states (x_k) and estimated parameters (w_k)), and the nonlinear mapping G is parameterized by w_k . The error of the mapping is $e_k = d_k - G(x_k, w_k)$, and the goal is to minimize the expected squared error. The d_k stands for the measured (desired) output. The estimation procedure for the parameter can be defined by the following expressions.

$$\begin{aligned}w_k &= w_{k-1} + u_k \\ d_k &= G(x_k, w_k) + e_k\end{aligned}\quad (2.70)$$

where w_k corresponds to a stationary process with an identity state transition matrix, driven by the noise u_k . The output y_k corresponds to the nonlinear observation on w_k .

The w_k parameters are updated via equation (2.65) for each k^{th} iteration. The distinguishing difference of UKF from other methods is that it uses sigma points and it is efficient, derivative-free for on-line learning.

2.3.2.3 UKF Parameter Estimation Procedure

The UKF addresses the problems faced in using of the EKF when applied to nonlinear systems. Using a carefully chosen set of sigma points, the UKF captures the true mean and covariance of the Gaussian Random Variables (GRV). A method for calculating the statistics of a random variable is the unscented transformation (UT). The UKF is a direct extension of the UT. First, one assumes the parameter (w_k) to be estimated has mean $\hat{w}_k$ and covariance P_{w_k} . To calculate the statistics of the output y_k in UKF, takes the form $y_{i,k/k-1}$, a matrix $W_{i,k/k-1}$ (with corresponding weights $W_i^{(m)}$) is formed, according to the following:

$$\begin{aligned} W_{i,k/k-1} &= \hat{w}_k & i &= 0 \\ W_{i,k/k-1} &= \hat{w}_k + (\sqrt{(\tilde{L} + \lambda)P_{w_k}})_i = \hat{w}_k + (\gamma\sqrt{P_{w_k}})_i \rightarrow i = 1 \dots, \tilde{L} \\ W_{i,k/k-1} &= \hat{w}_k - (\sqrt{(\tilde{L} + \lambda)P_{w_k}})_{i-\tilde{L}} = \hat{w}_k - (\gamma\sqrt{P_{w_k}})_i \rightarrow i = \tilde{L} + 1, 2\tilde{L} \end{aligned} \quad (2.71)$$

$$\begin{aligned} W_0^{(m)} &= \lambda / (\tilde{L} + \lambda) \\ W_0^{(c)} &= \lambda / (\tilde{L} + \lambda) + (1 - \alpha^2 + \tilde{\beta}) \\ W_i^{(m)} &= W_i^{(c)} = 1 / \{2(\tilde{L} + \lambda)\} \rightarrow i = 1, \dots, 2\tilde{L} \end{aligned} \quad (2.72)$$

where $\lambda = \alpha^2(\tilde{L} + \kappa) - \tilde{L}$ is a scaling factor. The α determines the spread of the sigma points ($W_{i,k/k-1}$) around $\hat{w}_k$ and is usually set to a small positive value (e.g. 0.001). The $\tilde{L}$ stands for the dimension of the parameter. The κ is an additional scaling factor and is usually set to zero. The $\tilde{\beta}$ is used to incorporate prior knowledge of the distribution of x

(for Gaussian distributions, $\tilde{\beta}=2$ is optimal). The $(\sqrt{(\tilde{L} + \lambda)P_w})_i$ is the i^{th} row of the matrix square root. The sigma points $(\mathbb{W}_{i,k/k-1})$ calculation is the checkpoint of whether the current parameter values are in the convergence range or not. This calculation is shown on Figures 2.8 and 2.9 via block diagram of the UT illustrating the generation of $\hat{d}_k$ and $P_{\hat{d}_k\hat{d}_k}$, and a flowchart of the UKF algorithm. The mean and covariance for output $\hat{d}_k$ are approximated using the weighting sample.

$$\hat{d}_k = \sum_{i=0}^{2\tilde{L}} W_i^{(m)} y_{i,k/k-1} \quad (2.73)$$

$$P_{\hat{d}_k\hat{d}_k} = \sum_{i=0}^{2\tilde{L}} W_i^{(c)} [y_{i,k/k-1} - \hat{d}_k][y_{i,k/k-1} - \hat{d}_k]^T + R_k^e \quad (2.74)$$

$$P_{w_k\hat{d}_k} = \sum_{i=0}^{2\tilde{L}} W_i^{(c)} [\mathbb{W}_{i,k/k-1} - \hat{w}_k][y_{i,k/k-1} - \hat{d}_k]^T$$

In UKF procedure, there are several important points to check to use this method [45]. First, the convergence rate and tracking performance are directly influenced by the choice of the covariance $E[u_k u_k^T] = R_k^r$. Units for R_k^r are the same as $[w_k w_k^T]$.

Basically, as R_k^r gets larger, the parameters are converging quickly. To choose R_k^r value, there are several options. One option is the randomly choosing a diagonal matrix. Another option is the Robbins-Monro stochastic approximation, which is

$$R_k^r = (1 - \alpha)R_{k-1}^r + \alpha K_k^w (d_k - G(x_k, \hat{w}))(d_k - G(x_k, \hat{w}))^T (K_k^w)^T \quad (2.75)$$

where R_k^r is also restricted to be a diagonal matrix. Wan et al [45] also found that a Robbins-Monro stochastic approximation method converges better, in general. Thus, in this dissertation, equation (2.75) was employed to calculate and update R_k^r . Secondly, complete convergence can be achievable when the covariance P_{w_k} goes to zero.

Determining P_{w_i} will produce a good basis for excellent convergence. In Chapter 3, the nature of the sigma points and demonstration of UKF convergence is further explained using the example of an inverted double pendulum.

2.4 Implementation Issues

From prior research, the base and moving platform joint locations and joint location errors were found to have an impact on the overall system dynamics (Johnston [35], Walker [39]). Since the main goal of this dissertation is estimating the dynamic parameters of the system understanding the different contributions of the different dynamic groups defined in equation (2.55) would provide insight as to which parameters may have a higher probability of being identifiable as well as those that may not be identifiable. This section presents a study of the above PKM dynamics grouped according to inertial and gravitational components for three different Stewart-Gough PKM configurations by comparing the relative magnitude of each term, an observation of the relative contribution to the dynamics response can be made. Based on this observation, a prediction of the dominant parameters can be made. In addition, this study provides insight as to the existence of trajectories that will yield sufficiently dynamically rich data necessary for parameter identification. The specific platforms included in the study are the University of Florida Special 6-6 PKM, the Hexel 6-3 PKM machine tool, and the Ford General 6-6 PKM vehicle simulator, shown in Figure 2.4 and described in section 2.1.1. Their analysis also provides insight to how geometry (e.g., joint locations) plays a role in the dynamics. Comparative observations are made using Euclidean norms and sample trajectory-based characterizations.

2.4.1 The Characterization of PKM Dynamics in Terms of the System Identification

2.4.1.1 System Model

In order to eliminate the overall PKM size effect on the results and to make a more direct comparison between the PKM variants presented in section 2.1.1 and Figure 2.4, their relative sizes were scaled to a physical size of the same order of dimensional scale as the UF-PKM. That is, the moving platforms of each PKM were scaled to form a theoretical equivalent equilateral triangle the same size as UF-PKM's moving platform. The Ford equivalent resulted in truncated corners of the equivalent triangle. All other dimensions maintained the same relative geometric spacing given by side:SIDE. The size of the UF PKM is $SIDE = 0.8128$ m and ratio side:SIDE = 0.750. The Hexel PKM has the joint configuration of the PKM shown in Figure 2.4 (b), size scaled to $SIDE = 0.4722$ m and ratio side:SIDE = 0.6096. Physically, it is constructed with platform joints paired in bifurcated ball joints where each strut is the screw of the ballscrew rigidly attached to the bifurcated ball joint at one end, and passing through the center of the base vertex (motor driven ball nut). The Ford PKM has a near octahedral layout similar to the Hexel, but with the platform joints in close pairs, as the base joints. The Ford PKM has the joint configuration of the PKM in Figure 2.4 (c), size scaled to $SIDE = 0.5276$ m and ratio side:SIDE = 1.12.

2.4.1.2 Trajectory Selection

In order to capture the dynamics, a motion must be specified. Unfortunately, the resulting dynamics is very dependent on the choice of motion. Hence, for the study presented here, various trajectories were selected. Sinusoidal translation trajectories of the platform in each of the X, Y, Z and X-Y directions about the 'home' position of the platform (center of its workspace at mid-stroke of struts) were selected and run for low,

medium and high velocities of the platform. Similarly, sinusoidal trajectories in rotational variables, Euler (ϕ, θ, ψ), of the platform about its home position were selected and run at 3 speed levels. For each of the above trajectories, a platform offset in the X-Y plane from the home position was also introduced for additional evaluation. The amplitudes of the motion were selected so that the PKM would remain within its physical workspace but span a reasonable breadth (50% or greater). Table 2.2 lists a subset of these trajectories by the Case Numbers used in Figures 2.10 through 2.15.

The Hexel and Ford PKMs were simulated for the same trajectory scaled up such that each machine moved from the center of their respective workspace to within 20% of the work boundary. Thus, while the UF PKM moved to a final position of $X=Y=0.04$ m, the Hexel PKM moved to $X=Y=0.3$ m and the Ford PKM moved to $X=Y=0.6$ m. Velocity and acceleration profiles were also scaled according to each PKMs maximum feed [UF $\rightarrow 0.3$ m/s (4 m/ sec²), Hexel $\rightarrow 0.3$ m/s (3 m/sec²), Ford $\rightarrow 1$ m/s (10 m/sec²)]. Tool orientations were 15 degrees for Hexel, 10 degrees for Ford, and 5 degrees for UF PKM..

2.4.1.3 Metrics of Identifiable Terms and Effort Expended

In order to establish which dynamics terms belong to an identifiable set, Euclidean norms of the F_a vector of actuator inputs and the model's individual $\mathbf{J}^{-1}[\mathbf{F}^T \quad \mathbf{T}^T]^T$ vectors (from equation (2.55)) were calculated along each of the above sinusoidal motion trajectories (listed in Table 2.2). By normalizing the Euclidean norms with respect to the mean value of the actuator F_a norm along a given trajectory, one achieves a 'relative' percentage measure of the theoretical dominance of each dynamics term in a corresponding measured F_a signal. Figures 2.10 through 2.15 provide a representative

set of the following norm evaluations along each given trajectory that was used for quantifying the results of this study. For Figures 2.10 through 2.13:

$$\text{pm norm} : \left\{ \text{mean} \left[\sqrt{\left(\mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix} \right)^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix}} \right] \div \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.76)$$

$$\text{sm norm} : \left\{ \text{mean} \left[\sqrt{\left(\mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix} \right)^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix}} \right] \div \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.77)$$

$$\text{cor norm} : \left\{ \text{mean} \left[\sqrt{\left(\mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{cor}^T & \mathbf{T}_{cor}^T \end{bmatrix} \right)^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{cor}^T & \mathbf{T}_{cor}^T \end{bmatrix}} \right] \div \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.78)$$

$$\text{cen norm} : \left\{ \text{mean} \left[\sqrt{\left(\mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{cen}^T & \mathbf{T}_{cen}^T \end{bmatrix} \right)^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{cen}^T & \mathbf{T}_{cen}^T \end{bmatrix}} \right] \div \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.79)$$

$$\text{grp norm} : \left\{ \text{mean} \left[\sqrt{\left(\mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{grp}^T & \mathbf{T}_{grp}^T \end{bmatrix} \right)^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{grp}^T & \mathbf{T}_{grp}^T \end{bmatrix}} \right] \div \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.80)$$

$$\text{grs norm} : \left\{ \text{mean} \left[\sqrt{\left(\mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{grs}^T & \mathbf{T}_{grs}^T \end{bmatrix} \right)^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{grs}^T & \mathbf{T}_{grs}^T \end{bmatrix}} \right] \div \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.81)$$

For Figures 2.14 and 2.15, the peak-to-peak value of the $\mathbf{F}_a$ norm along a given trajectory is calculated to provide a metric of the relative variations in the force where the larger values indicate a greater likelihood that parameters may be identifiable.

$$\text{Fa norm} : \left\{ (\text{max} - \text{min}) \text{ of } \left[\sqrt{F_a^T F_a} \right] \div \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.82)$$

$$\text{pm norm} : \left\{ (\text{max} - \text{min}) \text{ of } \left[\sqrt{\left(\mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix} \right)^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix}} \right] \div \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.83)$$

$$\text{sm norm} : \left\{ (\text{max} - \text{min}) \text{ of } \left[\sqrt{\left(\mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix} \right)^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix}} \right] \div \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.84)$$

$$\text{cor norm} : \left\{ (\text{max} - \text{min}) \text{ of } \left[\sqrt{\left(\mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{cor}^T & \mathbf{T}_{cor}^T \end{bmatrix} \right)^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{cor}^T & \mathbf{T}_{cor}^T \end{bmatrix}} \right] \div \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.85)$$

$$\text{cen norm} : \left\{ (\text{max} - \text{min}) \text{ of } \left[\sqrt{\left(\mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{cen}^T & \mathbf{T}_{cen}^T \end{bmatrix} \right)^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{cen}^T & \mathbf{T}_{cen}^T \end{bmatrix}} \right] \div \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.86)$$

$$\text{grp norm} : \left\{ (\text{max} - \text{min}) \text{ of } \left[\sqrt{\left(\mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{grp}^T & \mathbf{T}_{grp}^T \end{bmatrix} \right)^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{grp}^T & \mathbf{T}_{grp}^T \end{bmatrix}} \right] \div \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.87)$$

$$\text{grs norm} : \left\{ (\max - \min) \text{ of } \left[\sqrt{\left(\mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{gr}^T & \mathbf{T}_{gr}^T \end{bmatrix} \right)^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{F}_{gr}^T & \mathbf{T}_{gr}^T \end{bmatrix}} \right] + \text{mean} \left[\sqrt{F_a^T F_a} \right] \right\} * 100 \quad (2.88)$$

2.4.1.4 Dynamic Force Contributions

In this section, the PKMs shown in Figure 2.4 (a) are assumed to have the following mass parameters of the UF PKM: mass of platform $m_p = 4.82$ kg with $I_{xx} = I_{yy} = 0.0908$ kg-m² and $I_{zz} = 0.181$ kg-m². The strut masses are $m_b = 7.72$ kg with $I_{xxb} = I_{yyb} = 0.552$ kg-m² and $I_{z zb} = 0.00751$ kg-m², and $m_c = 1.79$ kg with $I_{xxc} = I_{yye} = 0.0183$ kg-m² and $I_{zzc} = 0.00160$ kg-m². The ratio $m_p/(6*m_c) = 0.45$. Using the prescribed trajectories of section 2.1, first paragraph, the corresponding contribution of the individual dynamic components for each PKM configuration is presented and discussed in section 2.1.1.

For the Hexel PKM, masses and inertias were estimated using dimensional and material data. The mass of the platform was estimated to be 93.0 kg with $I_{xx} = I_{yy} = 1.18$ kg-m² and $I_{zz} = 1.37$ kg-m². Strut masses were calculated as $m_b = 2.11$ kg with $I_{xxb} = I_{yyb} = 0.00135$ kg-m², and $m_c = 4.64$ kg with $I_{xxc} = I_{yye} = 0.556$ kg-m² and $I_{zzc} = 0.000362$ kg-m². The Hexel PKM ratio $m_p/(6*m_c) = 3.34$.

Most Ford PKM parameters were obtained from measurements or design specifications. In some cases, parameters had to be extrapolated from the given data. The values used in this study are as follows. Platform mass was 7500 kg with $I_{xx} = I_{yy} = 6.42 \times 10^3$ kg-m² and $I_{zz} = 1.28 \times 10^4$ kg-m². Strut masses were $m_b = 100$ kg with $I_{xxb} = I_{yyb} = 85.6$ kg-m² and $I_{z zb} = 0.500$ kg-m², and $m_c = 54.5$ kg with $I_{xxc} = I_{yye} = 1.83$ kg-m² and $I_{zzc} = 0.174$ kg-m². The Ford PKM ratio $m_p/(6*m_c) = 22.9$.

2.4.1.5 Identifiable metrics parameters

Implementing the series of Euclidean-based norms, a Matlab simulation of the PKM dynamics was run for each of the sinusoidal motions trajectories. The F_a mean (stands for $mean(\sqrt{F_a^T \cdot F_a})$) values were found to be relatively large for each motion case studied. Generally, these large means were attributed to the gravitational effects at low velocities ($0.0254 * \pi$ m/sec), while at higher platform velocities were attributed to the gravitational and the $\begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix}^T$ and/or $\begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix}^T$ effects. Figures 2.10 and 2.11 illustrate the relative contribution trend in the dynamics. The relative degree of contribution between the $\begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix}^T$ and $\begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix}^T$ shifts with the mass of the platform-to-mass of the struts ratio, $m_p/(6*m_c)$, as illustrated by Figures 2.12 and 2.13. As this mass ratio increases, $\begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix}^T$ relative dominance increases. The same trends were found for each PKM configuration of Figure 2.4, as well as for the Hexel and Ford PKMs. In terms of geometry affects, the PKMs of Figures 2.4 (b) and 2.4 (c) exhibited the same general trend shifts as those of PKM in Figure 2.4 (a). However examining Figures 2.11 and 2.13 closely, one also finds that as the joint locations approach the 6-3 configuration, the amount of the $\begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix}^T$ and $\begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix}^T$ contribution shift is amplified, particularly for $\begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix}^T$ in the horizontal and angular motion cases.

Figures 2.14 and 2.15 show the results of the force variation norms defined relative to the average of the F_a norm (equations (2.82) through (2.88)) for six of the motion cases studied. As can be seen from Figure 2.14, the variations of all forces were

relatively low for low platform velocities. Figure 2.15 illustrates that not only is the $\begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix}^T$ a dominant dynamic term based on the mean values (Figure 2.11), but truly dominates in terms of force variations relative to the $\mathbf{F}_a$ norm. The gravitational forces have small variations in their contributions to $\mathbf{F}_a$. For high platform velocities, the $\begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix}^T$ exhibits a modest contribution to the variations in $\mathbf{F}_a$. As the relative platform mass increases, the $\begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix}^T$ and $\begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix}^T$ switch roles. Although a very modest effect, Figure 2.14 delineates that offset in the motion generates greater variations in both the gravitational forces and $\mathbf{F}_a$.

Contrary to the above observations, the Hexel PKM exhibited relatively large $\mathbf{F}_a$ and $\begin{bmatrix} \mathbf{F}_{grp}^T & \mathbf{T}_{grp}^T \end{bmatrix}^T$ variation norms (20 to 58) for all motions studied where the motion had an offset from its center Z axis. This is attributed to the geometric effect of the small platform size relative to the base size, causing increased gravity loading variations within the struts.

To summarize and referring to Table 2.1, the selected motions of this study indicate that the identifiable set of dynamics terms for extendable strut PKMs will be $\begin{bmatrix} \mathbf{F}_{grs}^T & \mathbf{T}_{grs}^T \end{bmatrix}^T$, $\begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix}^T$, $\begin{bmatrix} \mathbf{F}_{grp}^T & \mathbf{T}_{grp}^T \end{bmatrix}^T$ and $\begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix}^T$. The Coriolis and centripetal dynamics terms were found to have very small effects in all cases. Hence, the dynamic parameters within these quantities may not be as easily identifiable, especially if the $\mathbf{F}_a$ signals are noisy. (Note, for PKMs that have significantly greater orientation capabilities this may not be the case.) In addition, changes in the $m_p/(6*m_c)$ ratio and the motion type and rates exhibit more apparent affects on the relative

dominance of the dynamic terms in a given trajectory than that observed for the changes in geometry. The above findings indicate that slow motions of the platform can be utilized to first identify the masses and the center of mass locations in a simplified, multiple step identification method. As the platform size relative to the base decreases, the identification of the other dynamic terms however may be more challenging. The horizontal motions (parallel to the base) are more suitable for identifying parameters within the $\begin{bmatrix} \mathbf{F}_{sm}^T & \mathbf{T}_{sm}^T \end{bmatrix}^T$ and $\begin{bmatrix} \mathbf{F}_{pm}^T & \mathbf{T}_{pm}^T \end{bmatrix}^T$ terms. While both vertical and slow motions are dominated by the $\begin{bmatrix} \mathbf{F}_{gs}^T & \mathbf{T}_{gs}^T \end{bmatrix}^T$ and $\begin{bmatrix} \mathbf{F}_{gp}^T & \mathbf{T}_{gp}^T \end{bmatrix}^T$ terms.

2.4.1.6 Summary

To summarize the findings of this section, an in-depth study on the dynamics has been conducted for three parallel kinematic mechanism configurations. From this study, the determination of which dynamics terms are ‘candidates’ for identification have been established, highlighting the potential limitations due to similarities in individual dynamic contributions and other system uncertainties. Consequently, the study indicates that dynamic system identification may require an ‘iterative’ methodology for system identification, so as to extract additional dynamic parameters. In brief, when $\mathbf{F}_{gp}$ dominates the signal, then one can expect to be able to extract the value of m_p easily. Similarly, for when $\mathbf{F}_{sm}$ dominates then one can expect to be able to extract $m_e, m_b, \mathbf{I}_e$ and $\mathbf{I}_b$.

Overall, the results show that trajectories sufficiently rich in dynamic excitation (commonly refer to as dynamically exciting trajectories) are plausible, hence the ability to identify the prominent dynamic parameters from experimental data exists.

2.4.2 The Parameter Sensitivity in Sensor Location

On-line parameter estimation has been shown to be a difficult task for the generalized Stewart PKM [33]. Therefore, in an effort to decrease the complexity and difficulties of actually implementing parameter identification methods, the effects of limited sensing instrumentation are studied in this section. The motivation of this study as well as the one just prior to this section was to provide a level of understanding and to illuminate areas of potential simplification necessary for realizing an industrial acceptance of the presented methodology. Thus, to simplify the estimation process, this section focuses on the validity of various simplifying assumptions. That is, the author suggests using the geometric center in lieu of estimating the true center of mass from the measured data for the sensor location.

2.4.2.1 Definition of Sensitivity

To identify the dynamic terms of special 6-6 PKM, it is required to experimentally measure the input torques and motion (e.g., motor current and encoder, respectively). Since the inaccuracy of the motor current causes significant noise and errors, new measurement device techniques are sought. Such techniques could consist of force/torque sensors placed directly on the platform or within the struts. For motion, the use of laser trackers or other optical devices are expensive and require unobstructed views. Hence, an alternative is to use motion sensors (for example, accelerometer, rate gyroscope) directly attached in the vicinity of center of mass of the moving platform to acquire acceleration, velocity and/or displacement as a function of time. For this case,

the accuracy of the location of the sensor could be the question. Hence the question in this section is, how sensitive is the calculated F_a to errors in center of mass locations estimates where the identification is run for estimating only mass and inertia parameters. Or, with limited sensing, use other measured data to calculate the center of mass motion. This research will examine the sensor location effect using the classical sensitivity analysis.

Using equation (2.55), one can identify the dynamic variables. In these dynamic terms, the location of the center of mass is chosen for checking sensitivity. Since the location of the center of mass and the orientation of the inertia tensors case both affect the accuracy of the identification of the dynamics, the importance of finding the exact location would be heightened. False measurement of the center of mass location may cause the vagueness of the results. In the simulation, the starting point of the center of the mass location (G) is same point as the geometric center location (C). Point P_i is location of joint i . However for the physical system, G will actually have an offset (r_{cg}) from C. Now, whether the offset vector r_{cg} plays a role for the sensitivity of the force variation is in question. To answer this, the classical dimensionless sensitivity equation (2.89) is used

$$S_{r_{cg}}^{F_{ai}} \cong \frac{\left(\Delta(F_a)_i / (F_a)_i \right)}{\left(\Delta(r_{cg})_i / (r_{cg})_i \right)} = \frac{\% \text{ change in } (F_a)_i}{\% \text{ change in } (r_{cg})_i} \quad (2.89)$$

where F_{ai} is the i^{th} component of the actuator force. This dimensionless equation was calculated using its numerical approximation. The $S_{r_{cg}}^{F_{ai}}$ represents the sensitivity when the offset (r_{cg}) exists.

2.4.2.2 Trajectory Selection and Conditions

For the trajectory condition, one needs to specify which direction to move. For this research, sinusoidal motions are selected (Table 2.3). Case numbers are defined for the different conditions and trajectory directions. Case numbers have two different trajectories. For pure translation case, the center of mass location moves into X_W direction with low frequency (0.5 Hz)

2.4.2.3 Sensitivity Results

The geometric center location of special 6-6 PKM is [0,0,-0.35] (meter). From this location, a one percent change of the maximum distance traveling to the X_W direction is chosen for the sensitivity evaluation. The results of the simulation are shown in Table 2.4. The case of $R_{cg} \neq 0$ and Beta_offset=0, the notation x#y#z# stands for the directional effect. The '0' means no change in that direction and the '1' means changes were made for that direction. The x0y1z1 means that there was no change in x direction and 1 percent change in both y and z directions. Note, the first row in each case corresponds to no design parameter variation, hence is always zero.

To interpret the results of equation (2.89) for each case shown in Table 2.4, a value of 5.5 in the table corresponds to a 5.5% to 1% ratio of change. Any value greater than one is considered significant since the design parameter variation effect is amplified. When R_{cg} is the selected design parameter, the offset effects of the center of mass are seen to be greatest in the terms of variations in the design parameters X and Z. Even during rotation, directional effects were of the same trend patterns.

To summarize this section's findings, an initial study of the sensitivity of the sensor location was made. The location offset was given as design sensitivity parameters. The

findings of this study indicate that the accuracy of the center of mass sensors potentially has a significant effect on the calculated actuator force (F_a) values depending on how large of error is in the location of the center of mass. Hence, simplification of the system identification algorithm may be limited in terms of accuracy of results desired. Chapter 5 presents the experimentally obtained identification results in which the center of masses were assumed known. Since the inertia estimates and center of mass locations for UF-PKM have been identified in prior research and the results readily available the assumptions is considered not too conservative nor hinders demonstrations of the UKF identification algorithms.

Table 2.1 Grouping of Terms by contributions to $\mathbf{JF}_a$

	Contains Terms		Contains Terms
$[F_{xy}^T \quad T_{xy}^T]^T$	$m_p \mathbf{g}$	$[F_{xz}^T \quad T_{xz}^T]^T$	All remaining m_c , m_b , I_c , and I_b terms not in $[F_{xyz}^T \quad T_{xyz}^T]^T$, $[F_{cor}^T \quad T_{cor}^T]^T$, $[F_{cen}^T \quad T_{cen}^T]^T$
$[F_{yz}^T \quad T_{yz}^T]^T$	$m_c \mathbf{g}$, $m_b \mathbf{g}$	$[F_{pbc}^T \quad T_{pbc}^T]^T$	F_{fric} , T_{fric}
$[F_{pm}^T \quad T_{pm}^T]^T$	$m_p \mathbf{a}_c$	$[F_{mb}^T \quad T_{mb}^T]^T$	$F_{m/b}$
$[F_{cor}^T \quad T_{cor}^T]^T$	$m_e \dot{L}_E(\omega_E \times \hat{\mathbf{s}}_3)$, $\dot{L}(I_E)\omega_E$ $m_b \dot{L}_B(\omega_B \times \hat{\mathbf{s}}_3)$, $\dot{L}(I_B)\omega_B$ $\dot{L}(\hat{\mathbf{s}}_3 \times \mathbf{v}_p) \times \hat{\mathbf{s}}_3$	$[F_{tool}^T \quad T_{tool}^T]^T$	F_{tool}
$[F_{cen}^T \quad T_{cen}^T]^T$	$m_e [\omega_E \times [\omega_E \times (L_E \hat{\mathbf{s}}_3)]]$ $I_e \hat{\mathbf{s}}_3 \times [\omega_c \times [\omega_c \times {}^w r_{pc}]]$ $\omega_c \times I_p \omega_c$, $\omega_B \times I_B \omega_B$ $\omega_E \times I_E \omega_E$, $m_e (\omega_E^T \omega_E) L \hat{\mathbf{s}}_3$ $m_e [\omega_B \times [\omega_B \times (L_B \hat{\mathbf{s}}_3)]]$ $I_B \hat{\mathbf{s}}_3 \times [\omega_c \times [\omega_c \times {}^w r_{pc}]]$ $m_b [\hat{\mathbf{s}}_3 \times [\omega_c \times [\omega_c \times {}^w r_{pc}]]] \times \hat{\mathbf{s}}_3$ $m_e [\hat{\mathbf{s}}_3 \times [\omega_c \times [\omega_c \times {}^w r_{pc}]]] \times \hat{\mathbf{s}}_3$	$[F_{umd}^T \quad T_{umd}^T]^T$	F_{umd} , T_{umd}

Table 2.2 Motion Trajectory Cases for Section 2.2

Motion Trajectory Cases			
Case Number	Trajectory		Radial Offset Xw-Yw Direction
1	Sinusoidal motion in X direction		0.0254 m
2	Sinusoidal motion in X direction		0.0127 m
3	Sinusoidal motion in Z direction		0 m
4	Sinusoidal motion in Z direction		0.0127 m
5	Sinusoidal (θ, ψ) rotational motion		5 degrees
6	Sinusoidal (θ, ψ) rotational motion		0.0127 m
Platform Motion Amplitudes	Low $\rightarrow$ Medium $\rightarrow$ High $\rightarrow$	Velocity: $(0.0254)*\pi$ m/sec Velocity: $(0.0254)*2\pi$ m/sec Velocity: $(0.0254)*4\pi$ m/sec	Acceleration: $(0.0254)*\pi^2$ m/sec ² Acceleration: $(0.0254)*4\pi^2$ m/sec ² Acceleration: $(0.0254)*16\pi^2$ m/sec ²
Frequency	Low $\rightarrow$ Medium $\rightarrow$ High $\rightarrow$	π rad/sec 2π rad/sec 4π rad/sec	

Table 2.3 Motion Trajectory Cases for Section 2.4

Case Number	Design Parameter Variation	Trajectory	Amplitude
1	$S_{r_{cg}}^{F_{a_i}}$	Pure Translation, X-direction	0.0254 m
2		Pure rotation	5 degree

Table 2.4 Table of Results for Section 2.4

Case	Trajectory	Mean Value Along Motion Trajectory										
		Condition	Strut 1	Strut 2	Strut 3	Strut 4	Strut 5	Strut 6				
$S_{t_{24}}^{F_{24}}$	Translate in x direction	x0y0z0	0.00	0.00	0.00	0.00	0.00	0.00				
		x0y0z1	-0.86	8.18	5.19	-3.96	-4.33	-4.23				
		x0y1z0	0.00	0.00	0.00	0.00	0.00	0.00				
		x0y1z1	-0.86	8.18	5.19	-3.96	-4.33	-4.23				
		x1y0z0	5.50	0.16	-2.01	-7.18	-3.50	7.01				
		x1y0z1	4.64	8.34	3.18	-11.13	-7.83	2.78				
		x1y1z0	5.50	0.16	-2.01	-7.18	-3.50	7.02				
		x1y1z1	4.64	8.34	3.18	-11.13	-7.83	2.78				
	Rotate 5 degrees	x0y0z0	0.00	0.00	0.00	0.00	0.00	0.00				
		x0y0z1	-0.86	8.20	5.18	-3.97	-4.31	-4.23				
		x0y1z0	0.00	0.00	0.00	0.00	0.00	0.00				
		x0y1z1	-0.86	8.20	5.18	-3.97	-4.31	-4.23				
		x1y0z0	5.48	0.15	-1.99	-7.17	-3.49	7.03				
		x1y0z1	4.62	8.35	3.19	-11.15	-7.80	2.80				
		x1y1z0	5.48	0.15	-1.99	-7.17	-3.49	7.03				
		x1y1z1	4.62	8.35	3.19	-11.15	-7.80	2.80				

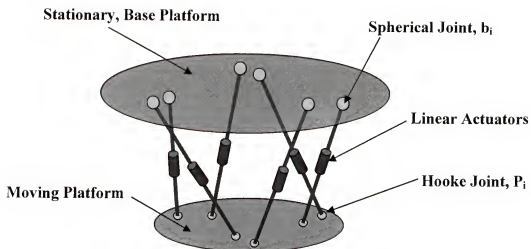


Figure 2.1 Schematic of a Stewart-Gough PKM

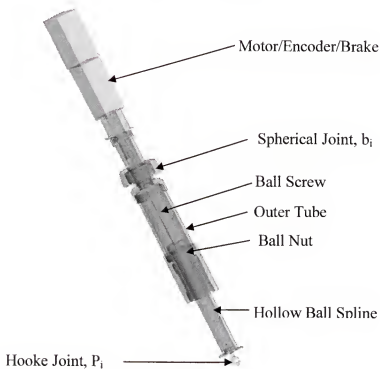


Figure 2.2 Physical Diagram of Strut i

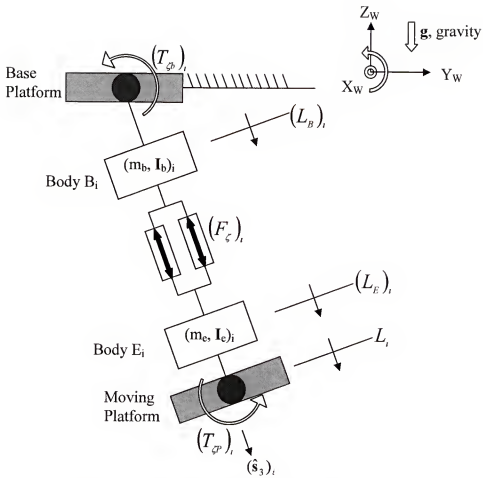
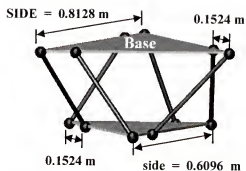
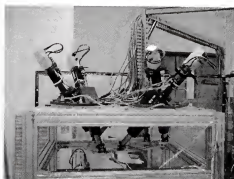
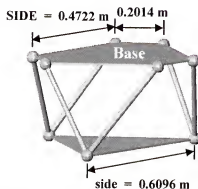


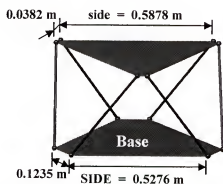
Figure 2.3 Lumped Mass Parameter Diagram for Strut i



(a) UF-SAMM, Special 6-6 PKM (side:SIDE=0.75)



(b) SNL Hexel, 6-3 PKM (side:SIDE=1.29)



(c) Ford Research Laboratory PKM, General 6-6 PKM (side:SIDE=1.12)

Figure 2 4 PKM Configuration Variants

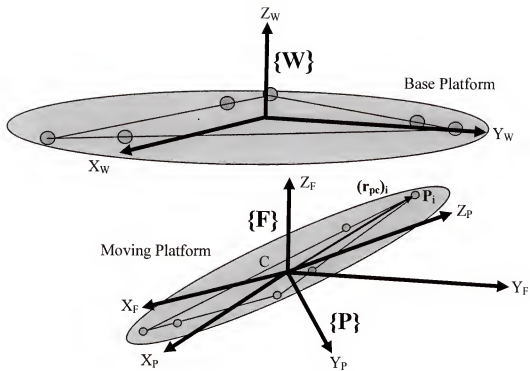


Figure 2.5 Coordinate System Assignment

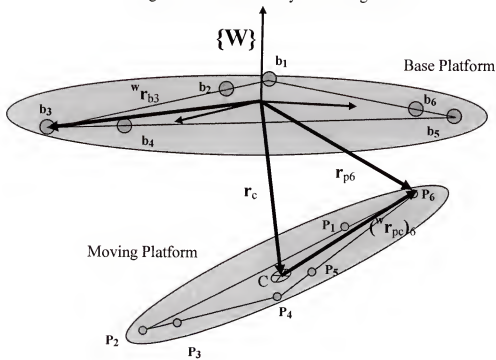


Figure 2.6 Naming Conventions

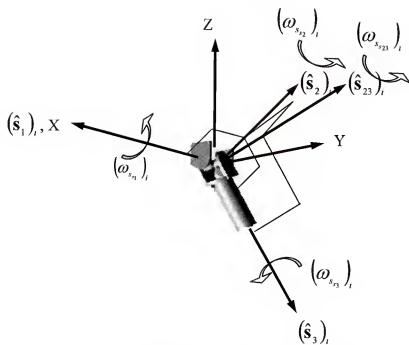


Figure 2.7 Strut Coordinate System

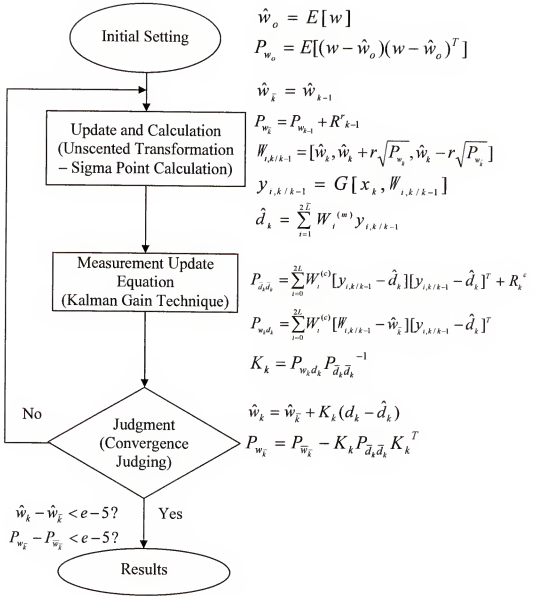


Figure 2.8 Flowchart for the UKF

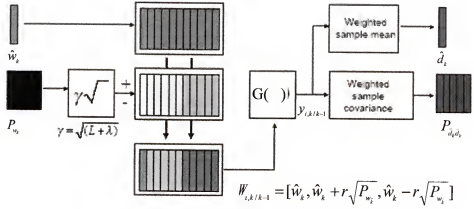


Figure 2.9 Block Diagram of Unscented Transformation [45]

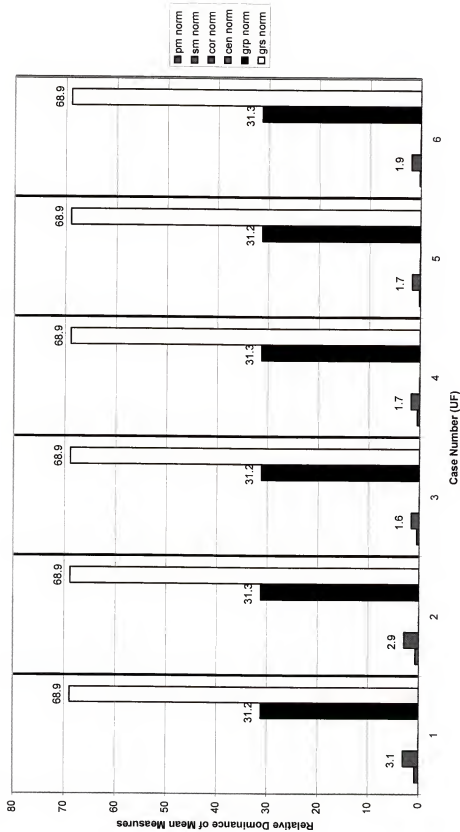


Figure 2.10 Results Defined by Equations 2.76 through 2.81 for Trajectories of Table 2.2 (Low Velocity Cases)

UF-PKM with side:SIDE=0.75 and Ratio $m_p/(6*m_c)=0.45$

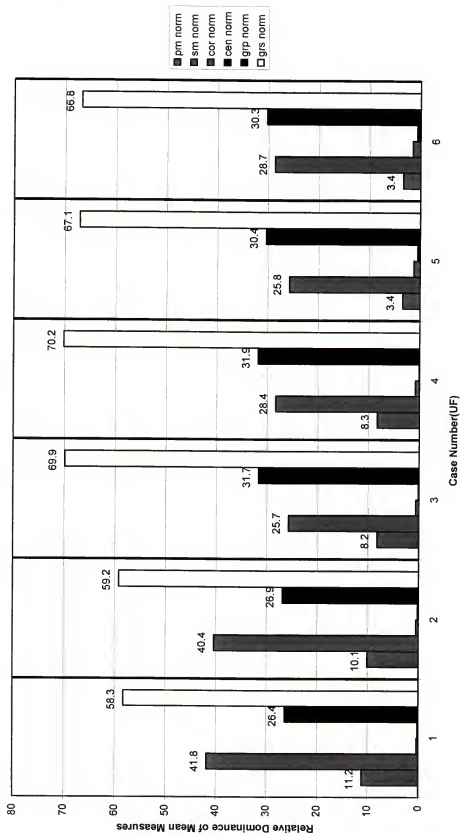


Figure 2.11 Results Defined by Equations 2.76 through 2.81 for Trajectories of Table 2.2 (High Velocity Cases)

UF-PKM with side:SIDE=0.75 and Ratio $m_p/(6*m_e)=0.45$

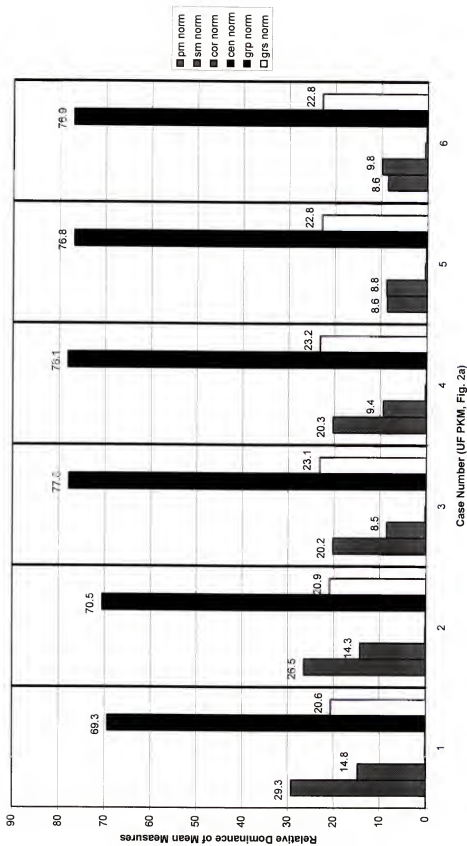


Figure 2.12 Results Defined by Equations 2.76 through 2.81 for Trajectories of Table 2.2 (High Velocity Cases)
 UF-PKM with side:SIDE=0.75 and Ratio $m_p/(6*m_c)=3.34$

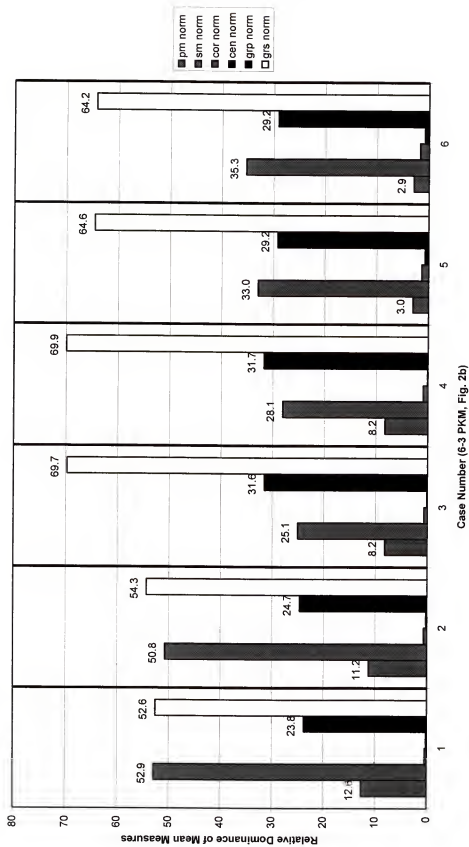


Figure 2.13 Results Defined by Equations 2.76 through 2.81 for Trajectories of Table 2.2 (High Velocity Cases)

6-3-PKM with side:SIDE=1.29 and Ratio $m_p/(6*m_c)=0.45$

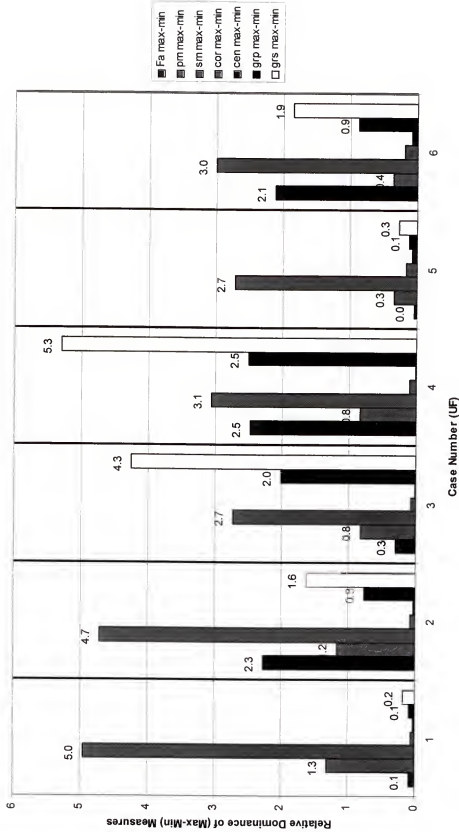


Figure 2.14 Results Defined by Equations 2.82 through 2.88 for Trajectories of Table 2.2 (Low Velocity Cases)

UF-PKM with side:SIDE=0.75 and Ratio $m_p/(6*m_c)=0.45$

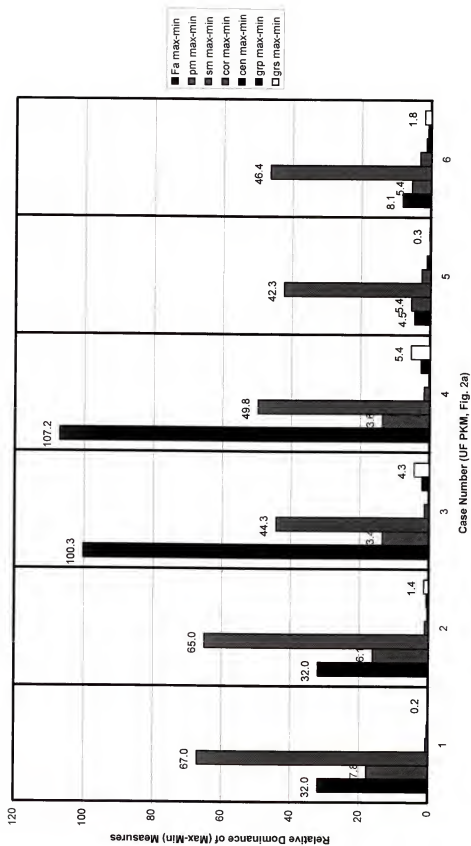


Figure 2.15 Results Defined by Equations 2.82 through 2.88 for Trajectories of Table 2.2 (High Velocity Cases)

UF-PKM with side:SIDE=0.75 and Ratio $m_p/(6*m_c)=0.45$

CHAPTER 3 IMPLEMENTATION OF PARAMETER IDENTIFICATION METHODS

This chapter illustrates the implementation of parameter identification methods and provides an evaluation of their performance. The inverted double pendulum example was selected as the case study due to its nonlinear dynamics and the availability of results by others in the literature. Simulation results delineate the effectiveness of the commonly used least squares (LS) method and the recently developed unscented Kalman filter (UKF) estimation method for estimating the dynamic parameters of nonlinear systems. The relative advantages and disadvantages of one method versus another are evaluated for determining each method's applicability in parameter identification of parallel mechanisms. This comparison is augmented with the results from the literature on the use of the extended Kalman filter (EKF). This chapter closes with the decision to use UKF for parameter identification of PKM dynamics. Implementation results for the PKM are presented in Chapters 4 and 5.

3.1 Pendulum Setup and Equations of Motion

An inverted double pendulum example has previously been used for comparing extended Kalman filter (EKF) and unscented Kalman filter (UKF) theory [38]. In this dissertation, the model and conditions of references [38, 45] are recreated for demonstrating the implementation of the UKF as well as adding the comparison of results obtained with the LS method. To make the simulation study more realistic, noise was added to the algorithm's simulated measured states (x_k); thus, emulating measurement

noise. In the inverted double pendulum example, noise was not added to the measured output (d_k) since the simulated system had zero driving forces and torques.

The inverted double pendulum (Figure 3.1) has the following states ($\mathbf{x}$) and parameters ($\mathbf{w}$)

$$\mathbf{x}=[x, \dot{x}, \theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2] \quad (3.1)$$

$$\mathbf{w}=[l_1, l_2, m_1, m_2, M] \quad (3.2)$$

where x : position of the cart

$\dot{x}$: velocity of the cart

θ_1 : angular position of the bottom pendulum, link 1 (absolute)

$\dot{\theta}_1$: rate of change in angular position of the bottom pendulum, link 1 (absolute)

θ_2 : angular position of the top pendulum, link 2 (absolute)

$\dot{\theta}_2$: rate of change in angular position of the top pendulum, link 2 (absolute)

l_1, m_1 : length and mass of the bottom pendulum (link 1), respectively

l_2, m_2 : length and mass of the top pendulum (link 2), respectively

M : mass of the cart

The mass of each link i is assumed to be uniformly distributed, where the center of mass is at $\frac{1}{2}l_i$. The gravitational force is assumed to be in the vertical direction as shown

in Figure 3.1. The dynamic equations of motion of this example are as follows.

$$\begin{aligned} (M + m_1 + m_2)\ddot{x} + \left(\frac{1}{2}m_1 + m_2\right)l_1\ddot{\theta}_1 \cos \theta_1 + \frac{1}{2}m_2l_2\ddot{\theta}_2 \cos \theta_2 \\ = F + \left(\frac{1}{2}m_1 + m_2\right)l_1\dot{\theta}_1^2 \sin \theta_1 + \frac{1}{2}m_2l_2\dot{\theta}_2^2 \sin \theta_2 \end{aligned} \quad (3.3)$$

$$\begin{aligned}
& \left(\frac{1}{2}m_1 + m_2\right)l_1\ddot{x}\cos\theta_1 + \left(\frac{m_1}{3} + m_2\right)l_1^2\ddot{\theta}_1 + \frac{1}{2}m_2l_1l_2\ddot{\theta}_2\cos(\theta_2 - \theta_1) \\
& = \left(\frac{1}{2}m_1 + m_2\right)gl_1\sin\theta_1 + \frac{1}{2}m_2l_1l_2\dot{\theta}_2^2\sin(\theta_2 - \theta_1)
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
& \frac{1}{2}m_2\ddot{x}l_2\cos\theta_2 + \frac{1}{2}m_2l_1l_2\ddot{\theta}_1\cos(\theta_2 - \theta_1) + \frac{1}{3}m_2l_2^2\ddot{\theta}_2 \\
& = \frac{1}{2}m_2gl_2\sin\theta_2 - \frac{1}{2}m_2l_1l_2\dot{\theta}_1^2\sin(\theta_2 - \theta_1)
\end{aligned} \tag{3.5}$$

The variable (F) is the control input force. For both the LS and UKF implementation, a ‘freefalling’ test setup was applied to obtain response data for the system state variables. The ‘freefalling’ test denotes generation of response (system state) data without any controller input applied to the system, i.e. only external load is gravity. In a physical implementation of an identification method, this data would be measured experimentally. However, to emulate the identification procedure, the system state data is generated using the ode45 solver in Matlab for the system as modeled in equations (3.3) thru (3.5), with the ‘true’ parameter values as known. The data collection time interval for the system is selected to be 0 second to 20 seconds where F is set to zero (no control input) for the entire simulation.

3.2 The Least Squares Method

To use the LS method, the reference residual ($e_i = (y_i - d_i)_i$, eqn (2.58)) must be defined. In this example, the d_i is the measured control input force and y_i is the corresponding calculated control input force for each k^{th} iteration using the assumed

theoretical model of the system given by equations (3.3) thru (3.5). I.e.,

$$y_k = \begin{bmatrix} F \\ 0 \\ 0 \end{bmatrix}_k = \begin{bmatrix} \left[\begin{array}{c} (M + m_1 + m_2)\ddot{x} + (\frac{1}{2}m_1 + m_2)l_1\ddot{\theta}_1 \cos \theta_1 + \frac{1}{2}m_2l_2\ddot{\theta}_2 \cos \theta_2 \\ -(\frac{1}{2}m_1 + m_2)l_1\dot{\theta}_1^2 \sin \theta_1 - \frac{1}{2}m_2l_2\dot{\theta}_2^2 \sin \theta_2 \end{array} \right] \\ \left[\begin{array}{c} (\frac{1}{2}m_1 + m_2)l_1\ddot{x} \cos \theta_1 + (\frac{m_1}{3} + m_2)l_1^2\ddot{\theta}_1 + \frac{1}{2}m_2l_1l_2\ddot{\theta}_2 \cos(\theta_2 - \theta_1) \\ -(\frac{1}{2}m_1 + m_2)gl_1 \sin \theta_1 - \frac{1}{2}m_2l_1l_2\dot{\theta}_2^2 \sin(\theta_2 - \theta_1) \end{array} \right] \\ \left[\begin{array}{c} \frac{1}{2}m_2\ddot{x}l_2 \cos \theta_2 + \frac{1}{2}m_2l_1l_2\ddot{\theta}_1 \cos(\theta_2 - \theta_1) + \frac{1}{3}m_2l_2^2\ddot{\theta}_2 \\ -\frac{1}{2}m_2gl_2 \sin \theta_2 + \frac{1}{2}m_2l_1l_2\dot{\theta}_2^2 \sin(\theta_2 - \theta_1) \end{array} \right] \end{bmatrix}_k \quad (3.6)$$

The d_k is set to $[0 \ 0 \ 0]$ (no control input force) for all time in the ‘freefalling’ test. The

corresponding $J_{LS,k}$ matrix from equation (2.61) is

$$J_{LS,k} = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} \end{bmatrix}_{w_{ls}} = \begin{bmatrix} \frac{\partial e_1}{\partial l_1} & \frac{\partial e_1}{\partial l_2} & \frac{\partial e_1}{\partial m_1} & \frac{\partial e_1}{\partial m_2} & \frac{\partial e_1}{\partial M} \\ \frac{\partial e_2}{\partial l_1} & \frac{\partial e_2}{\partial l_2} & \frac{\partial e_2}{\partial m_1} & \frac{\partial e_2}{\partial m_2} & \frac{\partial e_2}{\partial M} \\ \frac{\partial e_3}{\partial l_1} & \frac{\partial e_3}{\partial l_2} & \frac{\partial e_3}{\partial m_1} & \frac{\partial e_3}{\partial m_2} & \frac{\partial e_3}{\partial M} \end{bmatrix}_{w_{ls}} \quad (3.7)$$

where

$$J_{11} = (\frac{1}{2}m_1 + m_2)\ddot{\theta}_1 \cos \theta_1 - (\frac{1}{2}m_1 + m_2)\dot{\theta}_1^2 \sin \theta_1 \quad (3.7a)$$

$$J_{12} = \frac{1}{2}m_2\ddot{\theta}_2 \cos \theta_2 - \frac{1}{2}m_2\dot{\theta}_2^2 \sin \theta_2 \quad (3.7b)$$

$$J_{13} = \ddot{x} + \frac{1}{2}l_1\ddot{\theta}_1 \cos \theta_1 - \frac{1}{2}l_1\dot{\theta}_1^2 \sin \theta_1 \quad (3.7c)$$

$$J_{14} = \ddot{x} + l_1\ddot{\theta}_1 \cos \theta_1 + \frac{1}{2}l_2\ddot{\theta}_2 \cos \theta_2 - l_1\dot{\theta}_1^2 \sin \theta_1 - \frac{1}{2}l_2\dot{\theta}_2^2 \sin \theta_2 \quad (3.7d)$$

$$J_{15} = \ddot{x} \quad (3.7e)$$

$$J_{21} = \left(\frac{1}{2}m_1 + m_2\right)\ddot{x}\cos\theta_1 + 2\left(\frac{m_1}{3} + m_2\right)l_1\ddot{\theta}_1 + \frac{1}{2}m_2l_2\ddot{\theta}_2\cos(\theta_2 - \theta_1) \\ - \left(\frac{1}{2}m_1 + m_2\right)g\sin\theta_1 - \frac{1}{2}m_2l_2\dot{\theta}_2^2\sin(\theta_2 - \theta_1) \quad (3.7f)$$

$$J_{22} = \frac{1}{2}m_2l_1\ddot{\theta}_2\cos(\theta_2 - \theta_1) - \frac{1}{2}m_2l_1\dot{\theta}_2^2\sin(\theta_2 - \theta_1) \quad (3.7g)$$

$$J_{23} = \frac{1}{2}l_1\ddot{x}\cos\theta_1 + \frac{1}{3}l_1^2\ddot{\theta}_1 - \frac{1}{2}gl_1\sin\theta_1 \quad (3.7h)$$

$$J_{24} = l_1\ddot{x}\cos\theta_1 + l_1^2\ddot{\theta}_1 + \frac{1}{2}l_1l_2\ddot{\theta}_2\cos(\theta_2 - \theta_1) - \frac{1}{2}l_1l_2\dot{\theta}_2^2\sin(\theta_2 - \theta_1) - gl_1\sin\theta_1 \quad (3.7i)$$

$$J_{25} = 0 \quad (3.7j)$$

$$J_{31} = \frac{1}{2}m_2l_2\ddot{\theta}_1\cos(\theta_2 - \theta_1) + \frac{1}{2}m_2l_2\dot{\theta}_1^2\sin(\theta_2 - \theta_1) \quad (3.7k)$$

$$J_{32} = \frac{1}{2}m_2\ddot{x}\cos\theta_2 + \frac{1}{2}m_2l_1\ddot{\theta}_1\cos(\theta_2 - \theta_1) + \frac{2}{3}m_2l_2\ddot{\theta}_2 \\ - \frac{1}{2}m_2g\sin\theta_2 + \frac{1}{2}m_2l_1\dot{\theta}_1^2\sin(\theta_2 - \theta_1) \quad (3.7l)$$

$$J_{33} = 0 \quad (3.7m)$$

$$J_{34} = \frac{1}{2}l_2\ddot{x}\cos\theta_2 + \frac{1}{2}l_1l_2\ddot{\theta}_1\cos(\theta_2 - \theta_1) + \frac{1}{3}l_1^2\ddot{\theta}_2 - \frac{1}{2}gl_2\sin\theta_2 + \frac{1}{2}l_1l_2\dot{\theta}_1^2\sin(\theta_2 - \theta_1) \quad (3.7n)$$

$$J_{35} = 0 \quad (3.7o)$$

As stated in Section 2.3.1 (equation 2.64), parameter updates are calculated each iteration using

$$\Delta w_k = (J_{LS,k}^T J_{LS,k})^{-1} J_{LS,k}^T e_k \big|_{w_{k-1}} \quad (3.8)$$

$$w_{k+1} = w_k + \Delta w_k \quad (3.9)$$

where $e_k |_{w_{k0}}$ is the k^{th} reference residual evaluated at $w_k = w_{k0}$. I.e.,

$$e_k |_{w_{k0}} = (y_k - d_k) |_{w_{k0}} \quad (3.10)$$

The estimation is considered to have converged to true values when each element in $e_k |_{w_{k0}}$ has a value less than the acceptable tolerance. In this Chapter's simulations, a tolerance of 1×10^{-7} was used.

3.3 The Unscented Kalman Filter Setting

Since the UKF uses the original nonlinear equations, it does not need a linearization procedure. I.e., the original nonlinear equations of motion, (3.3)-(3.5), with zero control input yield

$$d_k = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.11)$$

and

$$G(x_k, w) = \begin{bmatrix} \begin{bmatrix} (M + m_1 + m_2)\ddot{x} + (\frac{1}{2}m_1 + m_2)l_1\ddot{\theta}_1 \cos \theta_1 + \frac{1}{2}m_2l_2\ddot{\theta}_2 \cos \theta_2 \\ -(\frac{1}{2}m_1 + m_2)l_1\dot{\theta}_1^2 \sin \theta_1 - \frac{1}{2}m_2l_2\dot{\theta}_2^2 \sin \theta_2 \end{bmatrix} \\ \begin{bmatrix} (\frac{1}{2}m_1 + m_2)l_1\ddot{x} \cos \theta_1 + (\frac{m_1}{3} + m_2)l_1^2\ddot{\theta}_1 + \frac{1}{2}m_2l_1l_2\ddot{\theta}_2 \cos(\theta_2 - \theta_1) \\ -(\frac{1}{2}m_1 + m_2)gl_1 \sin \theta_1 - \frac{1}{2}m_2l_1l_2\dot{\theta}_2^2 \sin(\theta_2 - \theta_1) \end{bmatrix} \\ \begin{bmatrix} \frac{1}{2}m_2\ddot{x}l_2 \cos \theta_2 + \frac{1}{2}m_2l_1l_2\ddot{\theta}_1 \cos(\theta_2 - \theta_1) + \frac{1}{3}m_2l_2^2\ddot{\theta}_2 \\ -\frac{1}{2}m_2gl_2 \sin \theta_2 + \frac{1}{2}m_2l_1l_2\dot{\theta}_2^2 \sin(\theta_2 - \theta_1) \end{bmatrix} \end{bmatrix} \quad (3.12)$$

As done in reference [45], all the parameters were initialized to 1.0. The other UKF searching values were selected as follows: $\alpha = 0.0001$, $\beta = 2$, $\kappa = 3 - n$, where for the inverted double pendulum example the dimension of w is $n=5$. The updating of R'_i or R^e_i uses Robbins-Monro Method (see section 2.3.2.3, equation 2.75).

3.4 Results

As can be seen in Table 3.1 for the LS estimate using the ‘without noise’ model, the l_1 and l_2 parameter values converged well to the true values, however the remaining parameter values converged only to within 15% error from the true value. A similar convergence trend can be found for the extended Kalman filter method [45] as shown in the bottom section of Table 3.1. For the LS method, a total of 1225 iteration steps were required to reach convergence within the allowable error tolerance (set to 1×10^{-7}). The total CPU time of this run was 4.23 seconds with a Pentium 4 (1.8 GHz), 512 M RAM computer. Figures 3.2 to 3.6 show a plot of the estimation results for each parameter at each iteration step of the LS method (dotted lines). The dashed line denotes the corresponding true parameter value and the solid line is the UKF results.

For the UKF estimate in Table 3.1, all parameters converged to true value within a 1 % error range in 2128 iterations and 68.88 CPU seconds for a tolerance setting of 1×10^{-7} . However, if one examines Figures 3.2 to 3.6, it is found that the UKF converged quickly to within a close range of the error tolerance but the program ran for approximately another 2000 iterations and took 68.88 CPU seconds before reaching the given error tolerance (1×10^{-7}). Thus, with a reduction in tolerance on the UKF convergence criterion, the CPU would be approximately the same as that for the LS

method (maintaining original tolerance). As seen in Table 3.1, the general trend in accuracy of parameter estimation in terms of tolerance for both LS and UKF methods showed a decrease in accuracy as the tolerance was loosened (e.g. tolerance $1 \times 10^{-7} \rightarrow 1 \times 10^{-3}$ resulted in accuracy decreasing by factors up to 22 times for UKF, yet still under 1% to 3.5% error).

Another comparison was made by adding noise in the system. To emulate the real system, random noise was added to the value generated from the data generating code described in Section 3.1 thus emulating measurement noise. The maximum value of this random noise was selected to be less than 1 % of the generated value. This correspondences to motion sensors with error specifications as follows: for x this corresponded to possible maximum error of 3 centimeters for a 3 meters range of motion; maximum error in $\dot{x}$ is 0.02 meters per second for range of 2.19 meters per second; maximum θ_1 is 0.36 radians for range of 36.5 radians; maximum $\dot{\theta}_1$ is 0.18 radians for range of 18.2 radians per second; maximum $\ddot{x}$ is 0.5 meters per second squared for range of 51.1 meters per second squared. The results of the LS and UKF methods subject to this random error are shown on Table 3.2 and Figures 3.7 through 3.11. The UKF estimation once again outperformed the LS estimation in terms of accuracy where both had the same number of iterations. While both estimation methods have merit, the LS method, however took less CPU time at the cost of requiring a priori linearization of the model base. For some parameters, it is observed that the measurement noise prevented premature termination of the LS estimation process, yielding slightly improved estimates. The EKF outperforms the LS but it too is limited by the need to linearize equations of motion before implementation. To conclude, although the UKF has a greater overhead in

CPU time, its greater convergence accuracy and ease of implementation for nonlinear systems (e.g., no linearization requirements) makes it the best candidate for dynamic parameter estimation of PKM's.

Table 3.1 Comparison Between UKF and LS Without Noise

	l_1 (meter)	l_2 (meter)	m_1 (kilogram)	m_2 (kilogram)	M (kilogram)	Iterations [Tolerance]	CPU Time (second)
True Parameter Value	0.50	0.75	0.75	0.50	1.50	---	---
LS Estimate (% Error)	0.5 (0.10)	0.75 (0.17)	0.64 (14.19)	0.42 (14.20)	1.28 (14.37)	1225	4.226
	0.50	0.75	0.74	0.50	1.49	2128	68.88
	(0.04)	(0.04)	(0.80)	(0.5)	(0.8)	$[1 \times 10^{-7}]$	
	(-0.08)	(-0.06)	(1.60)	(1.13)	(1.07)	154 $[1 \times 10^{-5}]$	5.42
UKF Estimate [45] (% Error)	(0.40)	(-0.86)	(2.08)	(-3.41)	(-1.15)	44 $[1 \times 10^{-3}]$	3.99
	0.50 (0.00)	0.75 (0.00)	0.68 (9.33)	0.45 (10.00)	1.35 (10.00)	N.A.	N.A.
UKF [45] (% Error)	0.50 (0.00)	0.75 (0.00)	0.75 (0.00)	0.50 (0.00)	1.49 (0.67)	N.A.	N.A.

Table 3.2 Comparison Between UKF and LS With Noise

	l_1	l_2	m_1	m_2	M	Iterations [Tolerance= 1×10^{-7}]	CPU Time (second)
True Parameter Value	0.50	0.75	0.75	0.50	1.50	---	---
UKF Estimate (% Error)	0.49 (1.70)	0.75 (-0.25)	0.78 (-3.87)	0.50 (0.72)	1.47 (1.93)	4257	302.16
LS Estimate (% Error)	0.49 (1.73)	0.73 (2.75)	0.66 (12.67)	0.44 (10.64)	1.27 (15.41)	4257	26.36

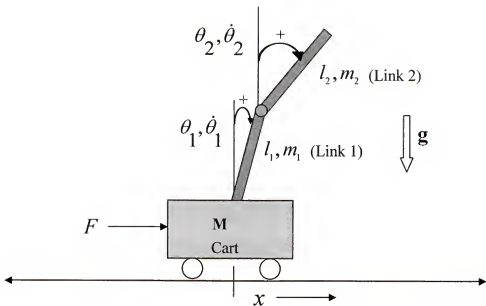


Figure 3.1 Double Inverted Pendulum

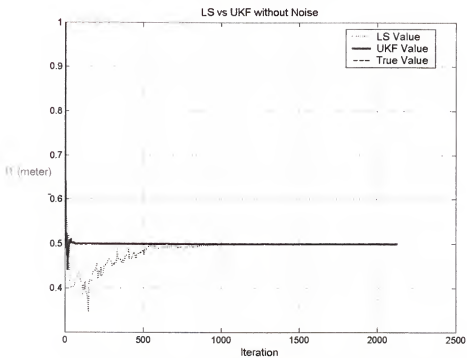


Figure 3.2 Value of l_1 , Case without Noise (LS vs. UKF)

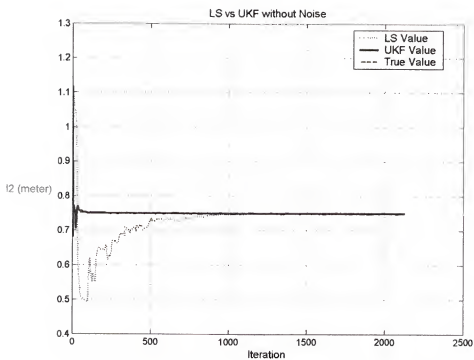


Figure 3.3 Value of I_2 , Case without Noise (LS vs. UKF)

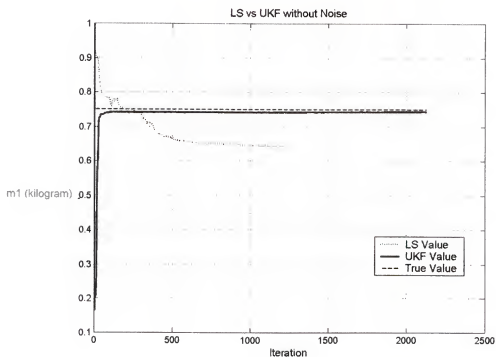


Figure 3.4 Value of m_1 , Case without Noise (LS vs. UKF)

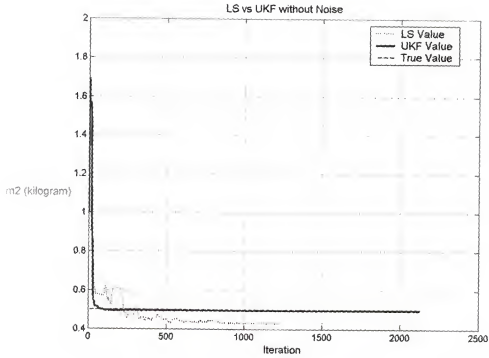


Figure 3.5 Value of m_2 , Case without Noise (LS vs. UKF)

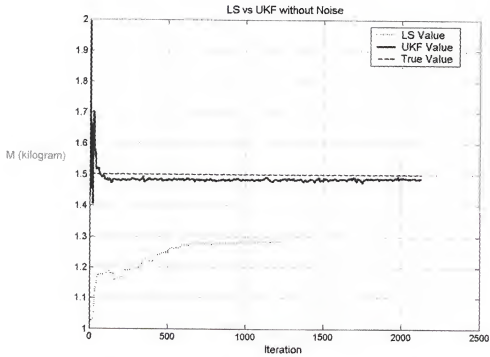


Figure 3.6 Value of M , Case without Noise (LS vs. UKF)

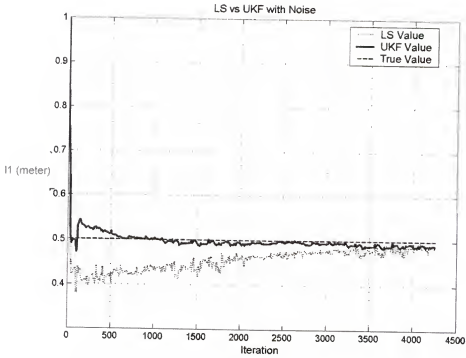


Figure 3.7 Value of l_1 , Case with Noise (LS vs. UKF)

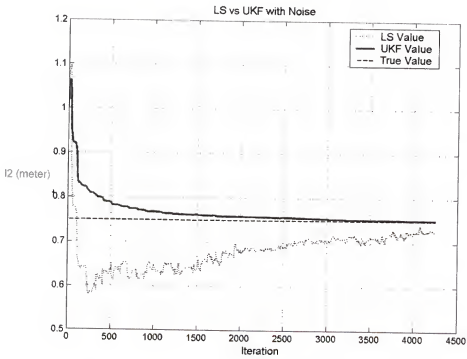


Figure 3.8 Value of l_2 , Case with Noise (LS vs. UKF)

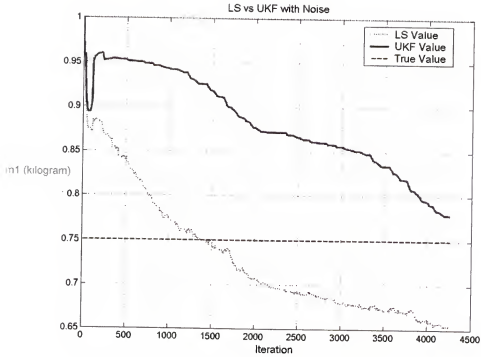


Figure 3.9 Value of m_1 , Case with Noise (LS vs. UKF)

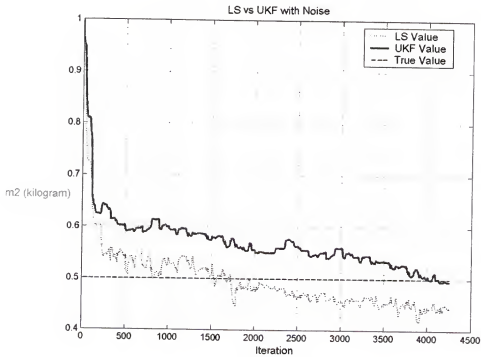


Figure 3.10 Value of m_2 , Case with Noise (LS vs. UKF)

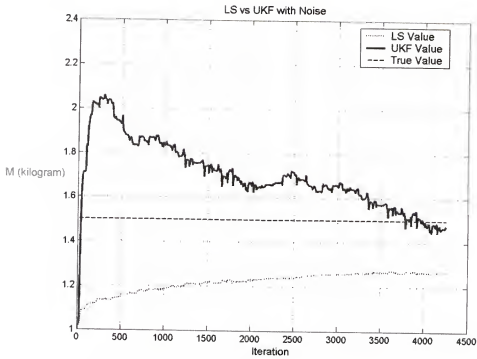


Figure 3.11 Value of M , Case with Noise (LS vs. UKF)

CHAPTER 4 IMPLEMENTATION OF UKF FOR PARAMETER IDENTIFICATION OF PKM DYNAMICS

This chapter presents a simulated implementation of the unscented Kalman filter (UKF) for parameter identification of PKM system parameters. The UF Special 6-6 PKM system was used to illustrate the performance of the identification algorithm in both simulated (Chapter 4) and experimental (Chapter 5) implementations. Implementation of the UKF parameter identification algorithm was done for two different forms of the PKM model base: force-based model (equation 2.55) and energy-based model (equation 2.56). The key difference between these two implementations is in the measured data, i.e. d_k desired output, and x_k measured states. The force-based model's measured data includes the actuator forces, and the platform's linear and angular positions, velocities and accelerations. The energy-based model's measured data includes the actuator forces, change in strut lengths, and the platform's linear and angular position and velocities. For the experimental implementation, the states were calculated from the measured strut lengths, time, and numerical derivatives determined from filtered link length measured data. The velocity and acceleration data can be either measured directly or determined from differentiated measured position data. The differentiation process can further exacerbate the problem of noise and/or measurement error by the introduction of an additional source of numerical errors. Since the data acquired from experiments always contains some form of noise and/or measurement errors, a comparative analysis of the UKF parameter identification algorithm's performance for the two different model-bases

was conducted to determine if there is any benefit in using the energy-based model approach other than reduced modeling effort in deriving the theoretical model. The anticipated benefit is reduced noise effects due to lower order derivatives and/or the number of measurements required in the energy-based method. If the results show a significant enhancement in performance, then a greater potential existed for the use of the UKF identification method for online estimation. For an industrial setting, online estimation can be the critical factor that potentially enables the engineer to further improve a machine's performance by reducing the operational time and machining errors.

4.1 PKM Simulations (Force-Based)

Referring to the PKM model presented in Chapter 2, the measured data in the force-based model are the actuator forces, and the platform's linear and angular positions, velocities and accelerations. To obtain the emulated measured actuator forces for a given state response, a simulation data-generating program of the model (equation 2.55) was used in which the emulated measured states were entered into the program as given. The emulated measured states are (refer to equations (2.4) through (2.8))

$$\mathbf{x}_k = (\mathbf{q}_k^T, \dot{\mathbf{q}}_k^T, \ddot{\mathbf{q}}_k^T)^T \quad (4.1)$$

The emulated measured (desired) output data is

$$\mathbf{d}_k = (\mathbf{F}_a)_{measured} \quad (4.2)$$

and the calculated output is

$$\mathbf{y}_k = G(\mathbf{x}_k, \mathbf{w}_k) \quad (4.3)$$

Again, it is noted that for implementation purposes, the center of mass locations were assumed known, i.e., not included in the set of parameters to be identified. In this dissertation, the CAD-based values that were determined by reference [48] are used. In

the analysis and implementation, it was also assumed that all struts are of modular design and have the same inertia parameters, thus reducing the number of parameters in the identification set. Hence, the parameters identified ($\mathbf{w}_k$) in the implementations are

$$\mathbf{w}_k = \left[m_e, m_b, m_p, I_{xxb}, I_{yyb}, I_{zzb}, I_{xxp}, I_{yyp}, I_{zyp}, I_{xxe}, I_{yye}, I_{zxe} \right]_k \quad (4.4)$$

As stated above, the data-generating program consists of the system model defined in Section 2.1 where the ‘true’ values for the inertia parameters (CAD-based values in this dissertation) were used for generating the emulated measured data. In this chapter, two different tests were conducted: freefalling test and actively controlled strut test. The freefalling test was used to generate emulated measured data as was done for the example problem in Chapter 3. The actively controlled strut test emulates an experimental implementation in which the strut forces are actively controlled. The freefalling test might be considered more useful in cases where the strut force data is very noisy or contains significant errors. After acquiring the emulated measured data from the data-generating program for each of these tests, the data was fed into the UKF algorithm for estimating the mass parameters. To further build confidence in the UKF parameter estimation method for PKMs, friction is introduced into the system and the results are compared to the frictionless case. As a representative model of friction found in mechanical systems, the modified friction model [46] was selected and is defined as follows.

$$F_{fric} = \begin{cases} Fric(i,1) \cdot \dot{L}_i + Fric(i,2) & \text{for } \dot{L}_i > 0.002 \\ (2 \cdot Fric(i,2) / 0.004) \cdot \dot{L}_i & \text{for } -0.002 < \dot{L}_i < 0.002 \\ Fric(i,1) \cdot \dot{L}_i - Fric(i,2) & \text{for } \dot{L}_i < -0.002 \end{cases} \quad (4.5)$$

where $Fric(i,1)$ = viscous friction coefficient for i^{th} strut

$Fric(i,2)$ = Coulomb friction coefficient for i^{th} strut

$\dot{L}_i$ = velocity of individual strut (Unit: m/s)

Table 4.1 provides the estimated coefficients for the friction model obtain using single strut tests of reference [37]. Figure 4.1 shows the vertical strut friction testbed and a representative plot of the results obtained for strut 1 [37].

4.2 PKM Simulations (Energy-Based Model)

The differences between the actual implementation of force-based model and energy-based model in UKF parameter identification are the model-base and the measured data. All other assumptions hold for both energy-based and force-based implementations. In the implementation cases presented, it is noted that the energy-based model results were for only the actively controlled strut tests. The energy-based model's data-generating program is the same as the one used for force-based model with an additional emulated measured data set of $(\Delta L_i)_k$ for each strut i and time step k . Hence, the emulated measured states for equation (2.56) are

$$\mathbf{x}_k = (\mathbf{q}_k^T, \dot{\mathbf{q}}_k^T)^T \quad (4.6)$$

and the emulated measured actuator ((desired) measured output) force data is

$$\mathbf{d}_k = \left(\sum_{i=1}^n (F_{a_i})_k (\Delta L_i)_k \right)_{measured} \quad (4.7)$$

$$\text{and } \mathbf{y}_k = h_{k+1}(\mathbf{x}_{k+1}, \mathbf{w}_{k+1}) - h_k(\mathbf{x}_k, \mathbf{w}_k) \quad (4.8)$$

where the parameters identified ($\mathbf{w}_k$) are the same set as used for force-based.

$$\mathbf{w}_k = [m_c, m_b, m_p, I_{xxb}, I_{yyb}, I_{zzb}, I_{xxp}, I_{yyp}, I_{zyp}, I_{xxc}, I_{yye}, I_{zzc}]_k \quad (4.9)$$

4.3 Simulation Setup

The same physical setup and conditions for both the force-based modeling and energy-based model implementations were used for each simulation case. For the freefalling test, the movable platform starts at the 'home' position (initial condition). The 'home' position is defined as the PKM configuration for which the mass center of the movable platform is located at $[x = 0.00, y = 0.00, z = -0.35]$ meters from the inertial frame $\{W\}$ as defined in Section 2.1.3.1. The physical dimensions of the PKM are as defined in Figure 2.4 (a).

For the actively controlled strut test, the movable platform starts at the 'home' position (initial condition). The mass center of the movable platform is then commanded to move sinusoidally about the 'home' position in the x-direction of the inertial frame with amplitude of one inch (case 1). It should be noted that the maximum width of UF-PKM workspace is approximately 4 inches in the X_w and Y_w plane. For case 2, the movable platform moves sinusoidally with the same amplitude about the 'home' position in the z-direction of the inertial frame. For case 3, the center of mass of the movable platform moves sinusoidally about the 'home' position in the positive/negative x and positive/negative y direction. The frequency of the movement is 4π Hz (approximately 45 % of the maximum speed capability of the UF PKM). The initial conditions of all motions are contained in Table 4.2. The trajectories (cases) used in the simulations and their parameter values are specified in Table 4.3

Again, note it has been assumed that all struts have the same inertia parameters and that the center of mass locations and kinematic parameters are known. The rule of notation used in the simulations are as follows:

- All headers and titles in the tables and figures relating to force-based model use the notation type: FTYPE1_TYPE2_TYPE3
 - For FTYPE1
 - FA denotes ‘active controlled strut tests’
 - FF for ‘freefalling tests’
 - Note that for FF type, there is no TYPE2.
 - For TYPE2
 - X for x-directional movement, Z for z-directional movement, and XY for xy-directional movement
 - For TYPE3
 - W indicates ‘with friction’ and WO represents ‘without friction’
- All notations relating to energy-based model follow the same rule as force-based model with the exception of using E instead of F for denoting the TYPE1 case.

4.4 Simulation Results (Force-Based, Without Friction)

Table 4.4 and 4.5 show the simulation results for the force-based model, without friction. For mass parameters, compared to the true value, the x-direction movement (FA_X_WO) and the xy direction movement (FA_XY_WO) yield the best estimates. The estimated mass values converged within 0.04% of its true value. The estimated mass parameters for z direction movement (FA_Z_WO) and freefalling case (FF_WO) however converged to within 16.5% and 11.4%, respectively. Table 4.5 shows the value of convergence for inertia parameters. In all cases, the movable platform parameter estimates (I_{xxp} , I_{yyp} , I_{zyp}) had better convergence than strut parameter estimates (I_{xxe} , I_{yye} , I_{zze} , I_{xxb} , I_{yyb} , $I_{z zb}$). The maximum percentage difference for platform parameters is 0.8%, however, the maximum percentage difference for strut parameters is 26.2%. The largest percentage difference can be seen on I_{yye} . This could be observed in two ways. One, the numerical value assigned in the simulation was very small compared to other parameter values (i.e., the largest parameter in the simulation was 7.7214, which is roughly 142,000 times greater than I_{yye} value (inertia about longitudinal axis of ball spline)). And/or, the selected test trajectories (data) do not yield sufficient dynamic excitation for

identification of the I_{yye} parameter. The dynamic excitation effects were explored further by randomizing or spreading the response data inputs ($\mathbf{x}_k, \mathbf{d}_k$). These effects will be discussed further in Section 4.9.

Figures 4.2-4.15 are plots of the results of the force-based, with and without friction. These plots are grouped into four categories: m_e (Figures 4.2 and 4.3), m_b (Figures 4.4 and 4.5), m_p (Figures 4.6 and 4.7), and all mass parameters (Figures 4.8-4.11). The plots in Figures 4.3, 4.5, 4.7 and 4.8-4.11 (without friction case) show a parameter convergence tendency towards their true values for the actively controlled strut cases (FA_X, FA_XY). For m_e case, the results of both x directional movement and xy directional movement converged faster and with greater accuracy than the other two (z and freefalling) trajectories. This tendency can also be found in other mass cases (m_b , m_p). For freefalling test, it shows generally poorer convergence accuracy for all cases. This can be attributed to the limitation of the strut and the workspace of the UF Special 6-6 PKM on its range of motion as well as the PKM configuration approaching a geometric condition in which struts are essentially parallel. That is, since the range of strut lengths is limited, the 'freefalling' movement has only a limited amount of data since the motion occurred too fast to capture in detail. This data was not sufficient to run the unscented Kalman filter algorithm for extended period without either reusing data sets or truncating the iterations. In this dissertation the iterations were truncated.

4.5 Simulation Results (Force-Based, With Friction)

Table 4.6 and 4.7 show the values of convergence for the force-based model, with friction results. For the mass properties, the estimated values are in error from the true values by less than 3% (x-direction), 2.6% (z-direction) and 16.2% (xy-direction). The

convergence error between the estimated value and true value in xy-direction exhibited a large increase compared to the without friction case. The results of the simulation for the 'with friction' case are shown for m_e case (Figure 4.2), m_b case (Figure 4.4), m_p case (Figure 4.6), and all mass parameters (Figures 4.12 through 4.15). For m_e case, z-direction case is the fastest and closest to the true value. The x-direction cases and xy-direction converge close enough to be considered good, but show slower convergence than the z-direction trajectory. Similar trends were found on m_b and m_p case.

4.6 Simulation Results (Energy-Based, Without Friction)

Comparing only the cases without friction, the energy-based model results are shown in Tables 4.8 and 4.9 for the estimated mass and inertia parameters. The plots showing parameter convergence tendencies are in Figures 4.16 thru 4.21. The resulting percentage differences between the true and estimated values all remained under 7.3 % for the mass properties. In general, the convergence performance rate and accuracy using energy-based model case was worse than that achieved using force-based model case with exception of the m_b estimate for the FA_Z_WO case. It is also observed in comparing energy-based results without friction with the force-based results with friction, the convergence accuracies exhibited to a large degree the same order of magnitude in percentage error. One notable difference (seen from results in Tables 4.4 and 4.8) is that the energy-based model achieved much better convergence accuracy, yielding a lower the percentage difference between true and estimated parameter value of I_{yye} . The percentage differences between true and estimated for the other inertia parameters are of either similar or higher magnitudes than that obtained using the force-based model. The energy-based model case was not done using the freefalling and the with friction tests,

hence assuming similar degrade in convergence accuracy would result as that found for the force-based model results.

The convergence tendencies are shown on Figures 4.16 (m_e), 4.17 (m_b), 4.18 (m_p). For m_e case, x and xy directional movement shows the similar trend of convergence. This trend can also be seen in m_b and m_p cases. The z-directional movement shows more of a linear tendency in convergence.

4.7 Simulation Results (Force-Based, Without Friction, Original/Random/Spread)

The k^{th} measured data $(\mathbf{x}_k, \mathbf{d}_k)$ consists of the system states and actuator forces measured at a given instant in time. In theory, given a perfect model-base and noise-free data, one could use a single set of measured data $(\mathbf{x}_k, \mathbf{d}_k)$ (i.e., $(\mathbf{x}_k, \mathbf{d}_k) = (\mathbf{x}_{k+1}, \mathbf{d}_{k+1})$ for all k) and run the UKF algorithm until $\mathbf{w}_k$ converges to the true estimate. Also, since $\mathbf{w}_k$ is an estimate of non-time varying parameters of the system, then any k^{th} set of measured data $(\mathbf{x}_k, \mathbf{d}_k)$ should yield the same $\mathbf{w}_k$ estimate. However, depending on the values of the measured states, it is possible that some $\mathbf{w}_k$ parameters would not be identifiable because at the given instant in time their contribution to the dynamics response is zero or essentially negligible. Thus, in practice, at each k^{th} iteration a new set of measured data $(\mathbf{x}_k, \mathbf{d}_k)$ is used so as to ensure the effects of all identifiable parameters are captured in the estimation process. The use of multiple data sets $(\mathbf{x}_k, \mathbf{d}_k)$ is also necessary when these sets are obtained from experiments, that in general contain noise and contributions from the unmodeled dynamics.

There are infinity possible data sets $(\mathbf{x}_k, \mathbf{d}_k)$ from which one can select. However there is no assurance, that the selected data sets will yield a $\mathbf{w}_k \rightarrow$ 'true' values, the

question arises as to which collection of data sets $(\mathbf{x}_k, \mathbf{d}_k)$ should one choose. That is, what motion trajectory should the PKM be driven to follow so as to yield a sufficiently dynamically exciting response from which to extract $(\mathbf{x}_k, \mathbf{d}_k)$. In search of a guarantee, researchers have developed various methodologies for generating sufficiently exciting trajectories [25]. As in reference [45], van der Merwe et al. used a free-fall approach. Per the study conducted in section 2.4, it was determined that sinusoidal trajectories about the “home” position provide sufficiently dynamically exciting data. However, to further substantiate the capabilities and limitations in using UKF and to explore the dynamic excitation effects, the force-based model UKF method was implemented where the response data time incremental sets were either fed into the algorithm in a random time order or simply in an increased spread in the time increment of the selected data. Figure 4.22 illustrates the difference in the ordering of the data sets for one variable input. The results in the prior sections used response time ordered $(\mathbf{x}_k, \mathbf{d}_k)$ data sets as depicted by the original case. To restate, since the time increments of the previous simulations from one time incremental step to the next were very small (0.001 second), the dynamic variation of the response data was limited as well throughout the estimation process. Thus, randomizing or spreading the $(\mathbf{x}_k, \mathbf{d}_k)$ data sequence was used as a means for validating that the prior results had sufficient dynamic excitation and to gauge how robust the UKF method. The term ‘random’ means that the sequence of the $(\mathbf{x}_k, \mathbf{d}_k)$ data sets given to the estimation algorithm is randomly chosen and the term ‘spread’ represents that the sequence of $(\mathbf{x}_k, \mathbf{d}_k)$ data sets given to the estimation

algorithm is chosen. e.g., from every 10 incremental steps (i.e. instead of using data number 1, 2, 3, ..., using 1, 11, 21, 31, ...).

For the force-based model without friction, Tables 4.10 and 4.11 show the results using randomized data. The percentage difference in x and xy direction shows good convergence. However, similar to the case using sequential data, the z-directional movement and freefalling tests do not yield high convergence. Tables 4.12 and 4.13 contain the results for 'spread' case. These results show better performance in the percentage difference compared to the 'random' case. Thus, the x and xy directional movements show superiority in convergence performance where the z direction exhibited the worst performance.

The plots that display the convergence of the 'random' and 'spread' case are in Figures 4.23 thru 4.42 where plots of the original estimation results (from Figures 4.8 through 4.11) are at times indistinguishable from the spread plots. That is, all figures relating to m_e , m_b , and m_p , respectively, in x direction and xy direction case present similar results. Data from the original (sequential) set and the spread case are comparable to each other, and the random case exhibits a slower convergence tendency. The z-direction case and freefalling cases do not have good convergence when using the data in the random nor the spread case, again experiencing the same limitations as cited earlier.

4.8 Comparison Between Force- and Energy-Based with Noisy Data

As an additional comparison between the force-based model and the energy-based model cases without friction, noise was added into the UKF estimation. The 1 % noise (per definition in Chapter 3) was added to the UKF measured data ($\mathbf{x}_k$) of the states. The

force-based model case was also ran with 5 % noise. As a representative case, the results for the xy-directional trajectory without friction simulation case is chosen, shown in Tables 4.14 through 4.17. These tables show that as the noise increases the difference between the estimates and the CAD-based ‘true’ values increases as expected. Figures 4.43 and 4.44 also support this observation. When comparing Tables 4.14 and 4.16, the mass parameters are estimated within 5.4 % for force-based model and 37.8 % for energy-based model when the 1% noise is added. This result shows that the force-based model outperforms the energy-based approach and hence is the preferred model-base to be used for the UKF. Additional convergence results for the other mass parameters are provided in Appendix C.

4.9 Summary of Results

Summarizing this Chapter provided comparisons on the selection of the model-base for UKF parameter identification in terms of estimation convergence accuracy and rate trends, for cases without friction, with different motion trajectories, and with and without noise. It was found that the x-directional case exhibited the lowest percentage difference between estimated and true parameter values. The xy-direction cases had similar trends for the x-direction cases; however, when friction was added to the system, the results were not as reliable as those found for the x-direction case. The convergence tendency for the z-directional movement and the ‘freefalling’ test were both poor, in general. Mostly, ‘freefalling’ tests suffered from the limited range of motion in which to capture a sufficient amount of data as well as the directional symmetry of the strut axes (also seen for the z-directional movement).

For the z-direction cases, energy-based model is preferred to the force-based model due to a lower percentage difference between the estimated and true values of the

relatively small parameters. However, for x-direction and xy-direction case, force-based model is superior to the energy-based model in terms of accuracy of convergence.

Hence, a significant improvement using energy-based method was not generally realized.

For data set types (original/random/spread), the results indicate that original data or spread data usage is the preferred for obtaining better estimation results, as well as it requires less data processing effort.

Table 4.1 Calculated Friction Value from Individual Strut Testing for 'Without Gearbox' Case

	Strut 1 *	Strut 2 **	Strut 3 **	Strut 4 *	Strut 5 **	Strut 6 **
Viscous (N*s/m)	1258 (333)	1012 (2681)	1099 (2901)	3517 (8930)	1140 (3023)	1168 (3100)
Coulomb (N)	50 (190)	49 (186)	58 (220)	131 (330)	49 (186)	53 (202)

Note, * Estimated friction coefficients obtained from horizontal single strut experiments.

** Estimated friction coefficients obtained from scaled reference [37] estimates

() stands for the friction value obtained from Reference [37]

Table 4.2 Initial Conditions

Position of the mass center of the movable platform	[0 0 -35] m
Velocity of the mass center of the movable platform	[0 0 0] m/sec
Acceleration of the mass center of the movable platform	[0 0 0] m/sec ²
Angular velocity of the mass center of the movable platform	[0 0 0] rad/sec
Angular acceleration of the mass center of the movable platform	[0 0 0] rad/sec ²

Table 4.3 Trajectories in Simulation (Four cases)

Case	1	2	3	4
Direction	x-direction	z-direction	xy-direction	Freefalling
Direction and magnitude of the movement	[0.0254 0 0] m	[0 0 0.0254] m	[0.0254 0.0254 0] m	Freefalling
Movement frequency of the mass center of the movable platform	4* π Hz	4* π Hz	4* π Hz	NA

Table 4.4 Mass Value (Units – kilogram)– Force-Based, Without Friction

Parameters	True Value	FA_X_WO	FA_X_WO % Diff	FA_Z_WO	FA_Z_WO % Diff	FA_XY_WO	FA_XY_WO % Diff	FF_WO	FF_WO % Diff
m _e	1.79	1.79	-0.00	1.80	-0.11	1.79	-0.00	1.62	8.65
m _b	7.72	7.72	0.04	6.46	16.47	7.72	0.04	6.89	10.49
m _p	4.82	4.82	0.01	4.91	-1.78	4.82	0.00	4.25	11.34

Table 4.5 Inertia Value (all times 1×10^{-6} , Units – Nm²) – Force-Based, Without Friction

Parameters	True Value (x10 ⁻⁶)	FA_X_WO	FA_X_WO % Diff	FA_Z_WO	FA_Z_WO % Diff	FA_XY_WO	FA_XY_WO % Diff	FF_WO	FF_WO % Diff
I _{xxb}	563500.00	563709.70	-0.04	510318.60	9.44	563709.70	-0.04	504657.10	10.44
I _{yyb}	69400.00	60508.83	12.81	60516.44	12.80	60508.83	12.81	60517.37	12.80
I _{zzb}	563500.00	503280.70	10.69	509689.80	9.55	503280.70	10.69	504657.30	10.44
I _{xxp}	90761.00	90041.92	0.79	90042.65	0.79	90041.92	0.79	90042.02	0.79
I _{yyp}	181283.60	180022.00	0.70	180023.30	0.70	180022.00	0.70	180022.00	0.70
I _{zpz}	90761.21	90025.98	0.81	90026.65	0.81	90025.98	0.81	90026.00	0.81
I _{xxe}	18269.40	18040.12	1.26	17771.91	2.72	18040.12	1.26	18039.45	1.26
I _{yye}	54.20	40.00	26.19	40.00	26.19	40.00	26.19	40.00	26.19
I _{zze}	18269.40	18045.12	1.23	18313.76	-0.24	18045.12	1.23	18045.98	1.22

Table 4.6 Mass Value (Units – kilogram)– Force-Based, With Friction

Parameters	True Value	FA_X_W	FA_X_W % Diff	FA_Z_W	FA_Z_W % Diff	FA_XY_W	FA_XY_W % Diff	FF_WO	FF_WO % Diff
m _e	1.79	1.78	0.83	1.78	0.79	1.67	7.07	1.55	13.86
m _b	7.72	7.50	2.92	7.86	-1.85	7.08	8.25	7.18	7.01
m _p	4.82	4.92	-1.97	4.94	-2.54	5.60	-16.11	4.69	2.80

Table 4.7 Inertia Value (all times 1x10⁻⁶, Units – Nm²) – Force-Based, With Friction

Parameters	True Value (x10 ⁻⁶)	FA_X_W	FA_X_W % Diff	FA_Z_W	FA_Z_W % Diff	FA_XY_W	FA_XY_W % Diff	FF_WO	FF_WO % Diff
I _{xxb}	563500.00	599181.80	-6.33	517260.50	8.21	543644.50	3.52	510242.50	9.45
I _{yyb}	69400.00	60511.60	12.81	60515.85	12.80	60505.87	12.82	60516.53	12.80
I _{zzb}	563500.00	503588.90	10.63	504016.90	10.56	543107.00	3.62	509748.30	9.54
I _{xyp}	90761.00	90041.95	0.79	90042.00	0.79	90036.97	0.80	90042.80	0.80
I _{yyp}	181283.60	180022.00	0.70	180022.00	0.70	180020.50	0.70	180023.60	0.70
I _{zyp}	90761.21	90025.99	0.81	90026.00	0.81	90023.17	0.81	90026.80	0.81
I _{xze}	18269.40	21865.73	-19.69	18040.25	1.25	19875.89	-8.79	17795.81	2.59
I _{yye}	54.20	40.00	26.19	40.00	26.19	40.40	25.44	40.00	26.19
I _{zze}	18269.40	18045.15	1.23	18045.15	1.23	20824.28	-13.98	18289.91	-0.11

Table 4.8 Mass Value (Units – kilogram)– Energy-Based, Without Friction

Parameters	True Value	EA_X_WO	EA_X_WO % Diff	EA_Z_WO	EA_Z_WO % Diff	EA_XY_WO	EA_XY_WO % Diff
m _e	1.79	1.84	-2.56	1.74	2.86	1.85	-3.24
m _b	7.72	7.92	-2.55	7.57	1.95	7.97	-3.25
m _p	4.82	4.55	5.71	4.50	6.48	4.47	7.29

Table 4.9 Inertia Value (all times 1×10^{-6} , Units – Nm²) – Energy-Based, Without Friction

Parameters	True Value	EA_X_WO	EA_X_WO % Diff	EA_Z_WO	EA_Z_WO % Diff	EA_XY_WO	EA_XY_WO % Diff
I _{xxb}	563500.00	564988.30	-0.27	526737.30	6.49	566573.40	-0.56
I _{yyb}	69400.00	60462.65	12.88	59948.68	13.65	60395.08	12.98
I _{zzb}	563500.00	514555.10	8.67	516902.50	8.21	565880.50	-0.44
I _{xxp}	90761.00	89992.58	0.84	89276.30	1.66	90034.70	0.78
I _{yyp}	181283.60	179863.30	0.78	178490.40	1.57	179963.80	0.71
I _{zzp}	90761.21	89953.00	0.89	89182.03	1.77	89852.50	1.00
I _{xxe}	18269.40	20455.38	-11.99	17877.57	2.18	20691.37	-13.30
I _{yye}	54.20	50.76	6.34	50.32	7.17	50.70	6.45
I _{zze}	18269.40	18091.38	0.94	17885.13	2.13	21222.95	-16.21

Table 4.10 Mass Value (Units – kilogram)– Force-Based, Without Friction Randomized case

Parameters	True Value	FA_X_WO	FA_X_WO % Diff	FA_Z_WO	FA_Z_WO % Diff	FA_XY_WO	FA_XY_WO % Diff	FF_WO	FF_WO % Diff
m _e	1.79	1.79	0.19	1.34	25.28	1.79	0.22	1.58	6.77
m _b	7.72	7.70	0.23	5.58	27.79	7.70	0.25	6.80	12.56
m _p	4.82	4.84	-0.42	7.51	-55.72	4.84	-0.48	4.34	11.80

Table 4.11 Inertia Value (all times 1x10⁻⁶, Units – Nm²) – Force-Based, Without Friction Randomized case

Parameters	True Value (x10 ⁻⁶)	FA_X_WO	FA_X_WO % Diff	FA_Z_WO	FA_Z_WO % Diff	FA_XY_WO	FA_XY_WO % Diff	FF_WO	FF_WO % Diff
I _{xxb}	563500.00	563183.30	0.06	512206.80	9.10	562322.80	0.21	504671.50	10.44
I _{yxb}	69400.00	60509.01	12.81	60509.34	12.81	60470.10	12.87	60517.37	12.80
I _{zzb}	563500.00	503300.00	10.68	512201.70	9.10	562213.90	0.23	504664.80	10.44
I _{xyp}	90761.00	90041.92	0.79	90041.93	0.79	90041.49	0.79	90042.02	0.79
I _{yyp}	181283.60	180022.00	0.70	180022.00	0.70	180021.90	0.70	180022.00	0.70
I _{zyp}	90761.21	90025.98	0.81	90025.98	0.81	90025.88	0.81	90026.00	0.81
I _{xze}	18269.40	18086.68	1.00	18040.13	1.26	18879.44	-3.34	18038.83	1.26
I _{yye}	54.20	40.00	26.19	40.00	26.19	39.98	26.22	40.00	26.19
I _{zze}	18269.40	18045.13	1.23	18045.13	1.23	18990.46	-3.95	18046.60	1.22

Table 4.12 Mass Value (Units – kilogram)– Force-Based, Without Friction Spread case

Parameters	True Value	FA_X_WO	FA_X_WO % Diff	FA_Z_WO	FA_Z_WO % Diff	FA_XY_WO	FA_XY_WO % Diff	FF_WO	FF_WO % Diff
m _e	1.79	1.79	-0.00	1.52	15.10	1.79	0.01	1.66	7.73
m _b	7.72	7.72	0.04	5.88	23.85	7.72	0.04	6.65	11.92
m _p	4.82	4.82	0.00	6.56	-36.16	4.82	-0.01	3.87	12.80

Table 4.13 Inertia Value (all times 1x10⁻⁶, Units – Nm²)– Force-Based, Without Friction Spread case

Parameters	True Value (x10 ⁻⁶)	FA_X_WO	FA_X_WO % Diff	FA_Z_WO	FA_Z_WO % Diff	FA_XY_WO	FA_XY_WO % Diff	FF_WO	FF_WO % Diff
I _{xxb}	563500.00	563707.50	-0.04	510407.00	9.42	563693.40	-0.03	504662.70	10.44
I _{yyb}	69400.00	60508.83	12.81	60511.26	12.81	60454.66	12.89	60517.37	12.80
I _{zzb}	563500.00	503280.70	10.69	510402.00	9.42	563688.40	-0.03	504651.40	10.44
I _{xyp}	90761.00	90041.92	0.79	90041.95	0.79	90041.32	0.79	90042.02	0.79
I _{ryp}	181283.60	180022.00	0.70	180022.00	0.70	180021.80	0.70	180022.00	0.70
I _{zrp}	90761.21	90025.98	0.81	90025.99	0.81	90025.84	0.81	90026.00	0.81
I _{xxe}	18269.40	18040.12	1.26	18040.15	1.26	18039.51	1.26	18039.86	1.26
I _{rye}	54.20	40.00	26.19	40.00	26.19	39.98	26.23	40.00	26.19
I _{zze}	18269.40	18045.12	1.23	18045.15	1.23	18044.55	1.23	18045.57	1.22

Table 4.14 Mass Values with Noisy Data (Units – kilogram)– Force-Based, Without Friction (0, 1 and 5 % Noise)

Parameters	True Value	FWO XY 0% Noise	% diff from CAD	FWO XY 1% Noise	% diff from CAD	FWO XY 5% Noise	% diff from CAD
m _e	1.79	1.79	0.04	1.75	2.37	1.66	7.57
m _b	7.72	7.72	0.07	7.56	2.11	7.47	3.25
m _p	4.82	4.82	-0.08	5.08	-5.40	5.74	-19.09

Table 4.15 Inertia Values with Noisy Data (all times 1×10^{-6} , Units – Nm²) – Force-Based, Without Friction (0, 1 and 5 % Noise)

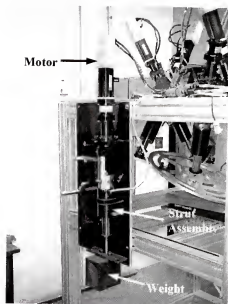
Parameters	True Value	FWO XY 0% Noise	% diff from CAD	FWO XY 1% Noise	% diff from CAD	FWO XY 5% Noise	% diff from CAD
I _{xxb}	563500.00	560986.30	0.45	552199.60	2.01	518173.10	8.06
I _{yyb}	69400.00	60509.69	12.81	60510.56	12.81	60515.68	12.80
I _{zzb}	563500.00	560025.90	0.61	503473.50	10.65	504044.10	10.55
I _{xxp}	90761.00	90041.93	0.79	90041.94	0.79	90042.00	0.79
I _{yy p}	181283.60	180022.00	0.70	180022.00	0.70	180022.00	0.70
I _{zzp}	90761.21	90025.98	0.81	90025.99	0.81	90026.00	0.81
I _{xxe}	18269.40	20703.04	-13.31	18039.90	1.26	18037.62	1.27
I _{yye}	54.20	40.00	26.19	40.00	26.19	40.00	26.19
I _{zze}	18269.40	21615.76	-18.32	18045.14	1.23	18045.20	1.23

Table 4.16 Mass Values with Noisy Data (Units – kilogram)– Energy-Based,Without Friction (0 and 1 % Noise)

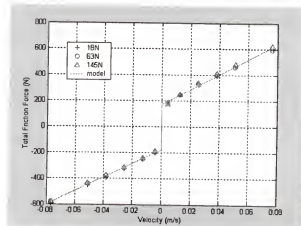
Parameters	True Value	EW OXY 0% Noise	% diff from CAD	EW OXY 1% Noise	% diff from CAD
m _e	1.79	1.85	-3.24	2.10	-17.19
m _b	7.72	7.97	-3.25	9.06	-17.40
m _p	4.82	4.47	7.29	3.01	37.79

Table 4.17 Inertia Values with Noisy Data (all times 1×10^{-6} , Units – Nm²) – Energy-Based, Without Friction (0 and 1 % Noise)

Parameters	True Value	EW OXY 0% Noise	% diff from CAD	EW OXY 1% Noise	% diff from CAD
I _{xxb}	563500.00	566573.40	-0.56	597686.70	-6.09
I _{yyb}	69400.00	60395.08	12.98	60309.58	13.10
I _{zzb}	563500.00	565880.50	-0.44	596866.50	-5.94
I _{xyp}	90761.00	90034.70	0.78	89907.69	0.92
I _{yyp}	181283.60	179963.80	0.71	179716.10	0.85
I _{zyp}	90761.21	89852.50	1.00	89725.26	1.14
I _{xze}	18269.40	20691.37	-13.30	22626.75	-23.90
I _{yze}	54.20	50.70	6.45	50.63	6.58
I _{zze}	18269.40	21222.95	-16.21	23158.06	-26.81

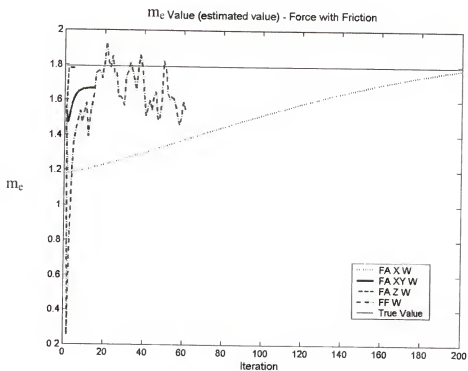


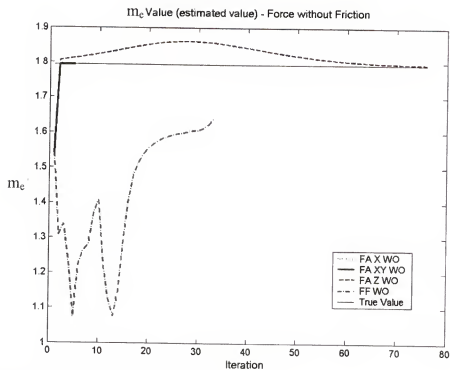
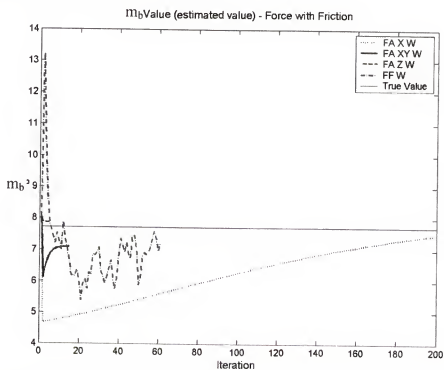
(a) The Vertical Strut Friction Test Stand

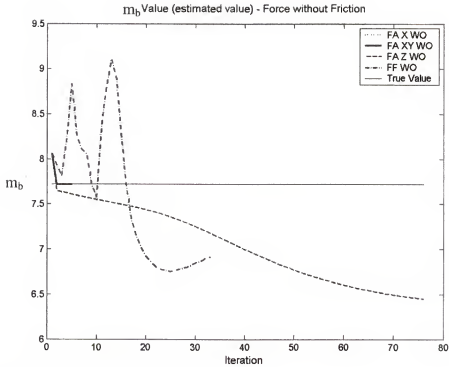
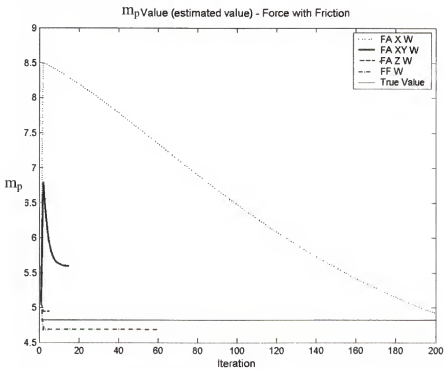


(b) Strut 1 Friction Data

Figure 4.1 Single Strut Friction Test [37]

Figure 4.2 m_e Value (Force with Friction)

Figure 4.3 m_c Value (Force without Friction)Figure 4.4 m_b Value (Force with Friction)

Figure 4.5 m_b Value (Force without Friction)Figure 4.6 m_p Value (Force with Friction)

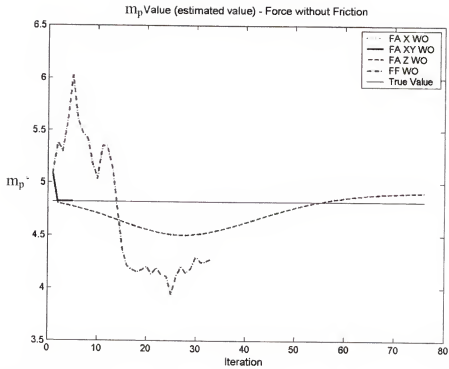
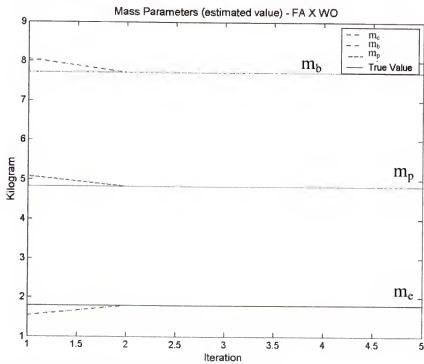
Figure 4.7 m_p Value (Force without Friction)

Figure 4.8 Mass Values (X-direction without Friction)

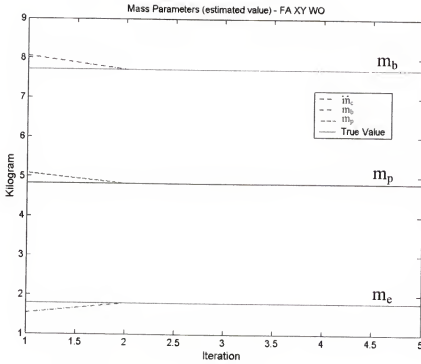


Figure 4.9 Mass Values (XY-direction without Friction)

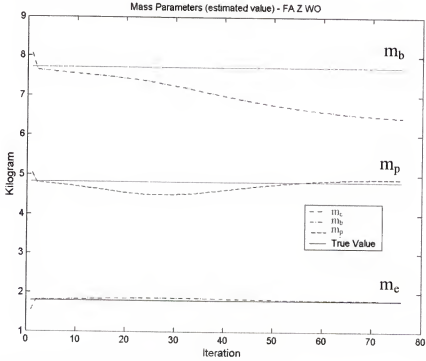


Figure 4.10 Mass Values (Z-direction without Friction)

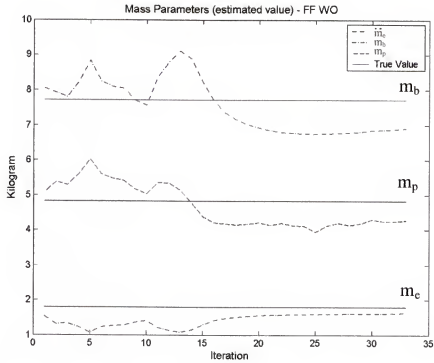


Figure 4.11 Mass Values (FF case without Friction)

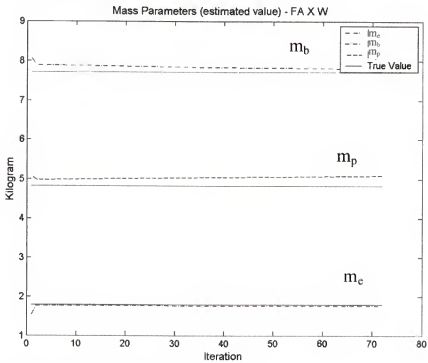


Figure 4.12 Mass Values (X-direction with Friction)

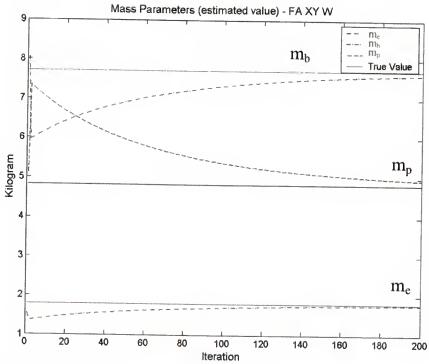


Figure 4.13 Mass Values (XY-direction with Friction)

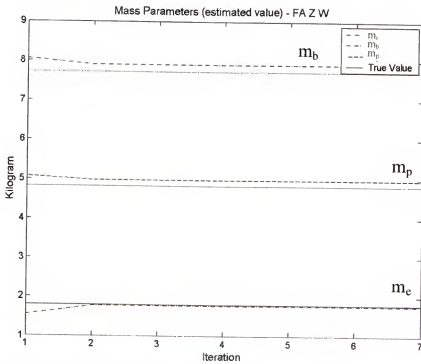


Figure 4.14 Mass Values (Z-direction with Friction)

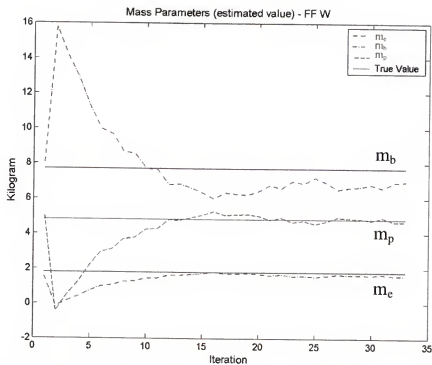
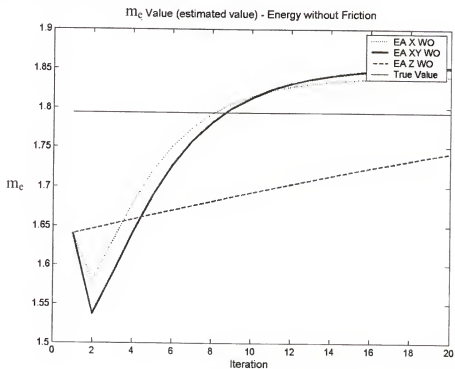
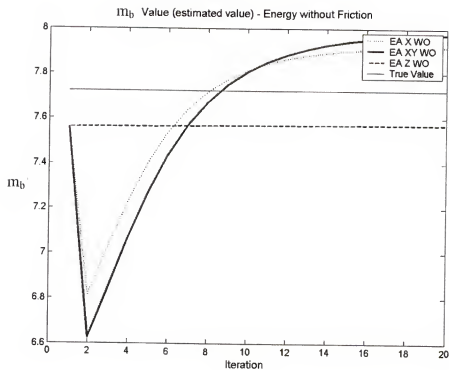
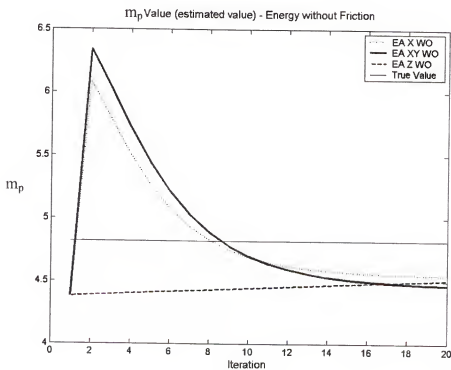


Figure 4.15 Mass Values (FF case with Friction)

Figure 4.16 m_e Value (Energy without Friction)

Figure 4.17 m_b Value (Energy without Friction)Figure 4.18 m_p Value (Energy without Friction)

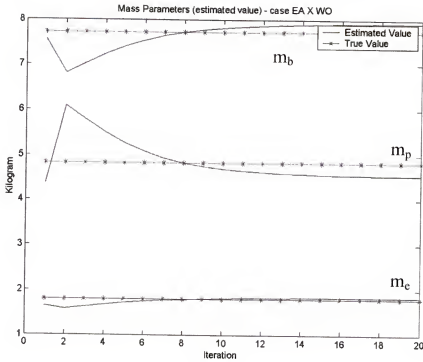


Figure 4.19 Mass Values (X-direction: Energy-Based without Friction)

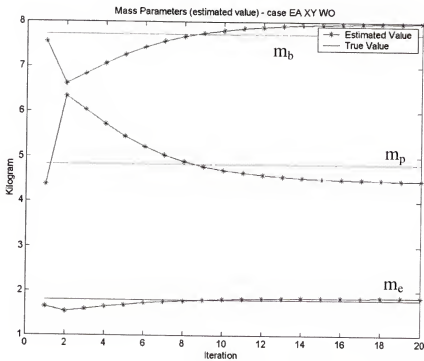


Figure 4.20 Mass Values (XY-direction: Energy-Based without Friction)

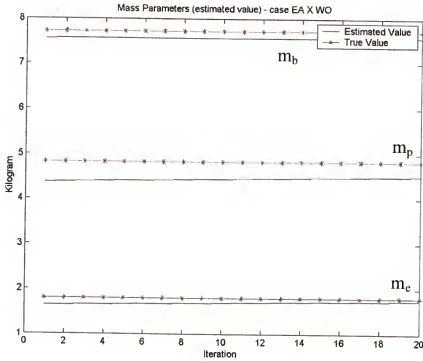


Figure 4.21 Mass Values (Z-direction: Energy-Based without Friction)

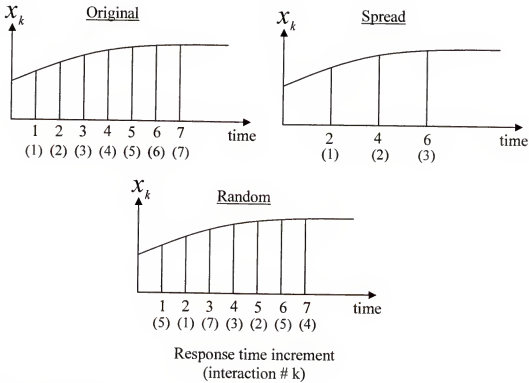


Figure 4.22 Data Set Collection Type (Original, Spreading and Randomizing)

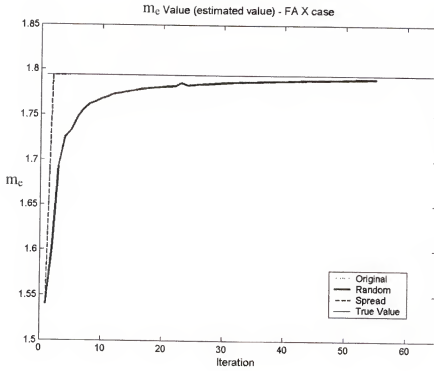


Figure 4.23 m_c Value (FA X WO – Original/Random/Spread)

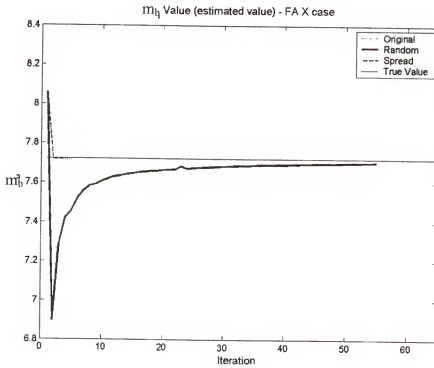


Figure 4.24 m_b Value (FA X WO – Original/Random/Spread)

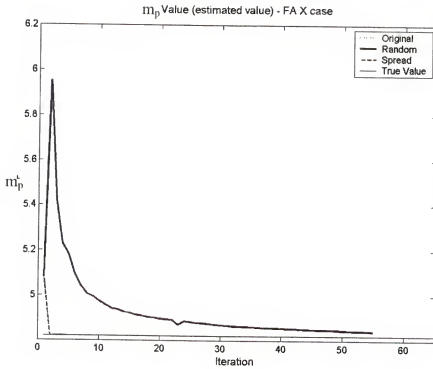


Figure 4.25 m_p Value (FA X WO – Original/Random/Spread)

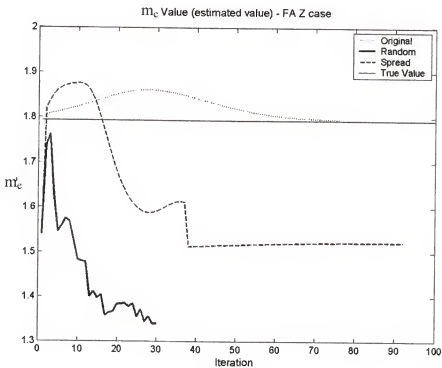


Figure 4.26 m_c Value (FA Z WO – Original/Random/Spread)

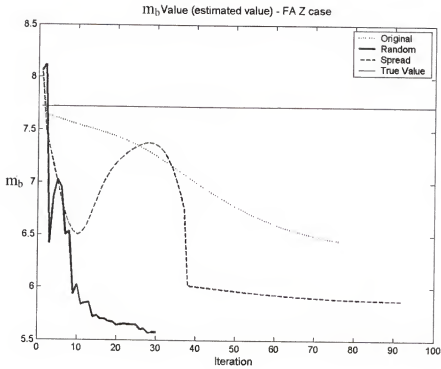


Figure 4.27 m_b Value (FA Z WO – Original/Random/Spread)

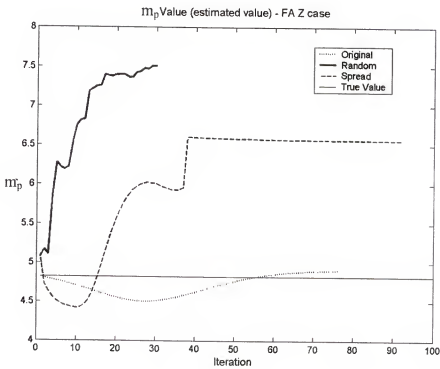


Figure 4.28 m_p Value (FA Z WO – Original/Random/Spread)

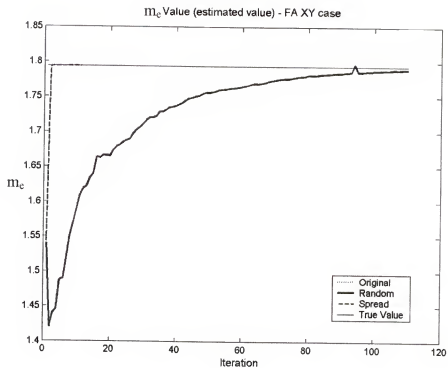


Figure 4.29 m_c Value (FA XY WO – Original/Random/Spread)

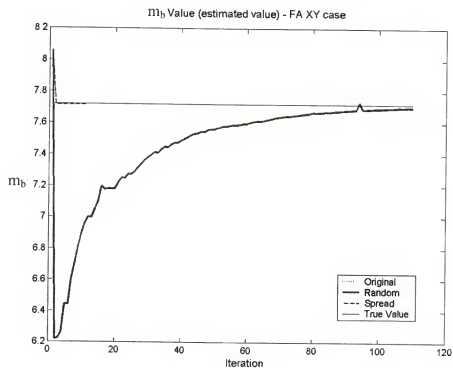


Figure 4.30 m_b Value (FA XY WO – Original/Random/Spread)

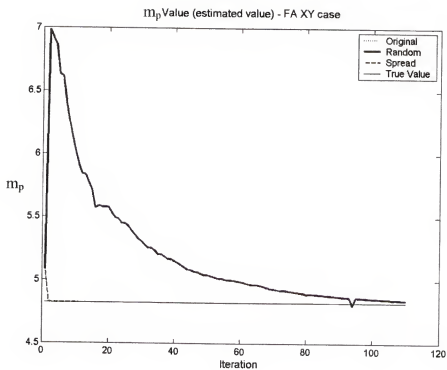


Figure 4.31 m_p Value (FA XY WO – Original/Random/Spread)

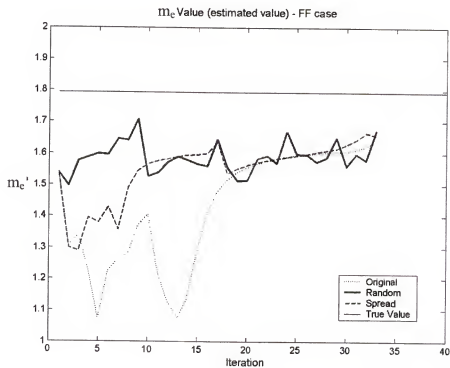


Figure 4.32 m_c Value (FF WO – Original/Random/Spread)

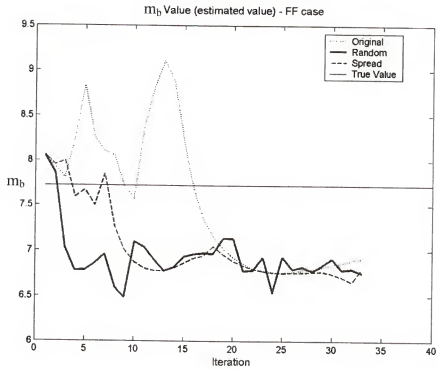


Figure 4.33 m_b Value (FF WO – Original/Random/Spread)

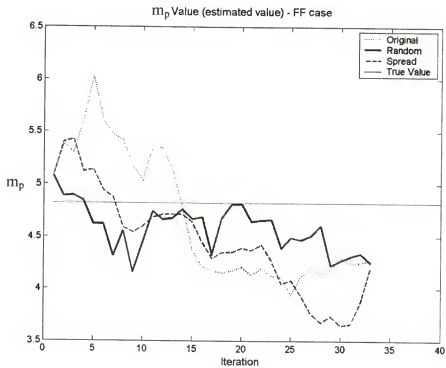


Figure 4.34 m_p Value (FF WO – Original/Random/Spread)

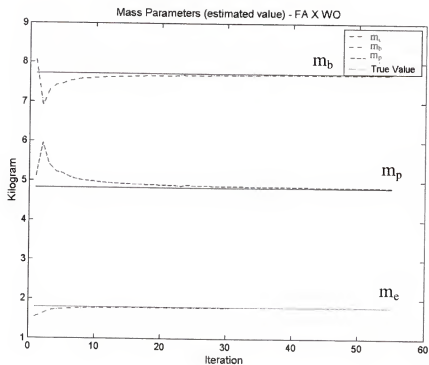


Figure 4.35 Mass Value (X-direction, Random)

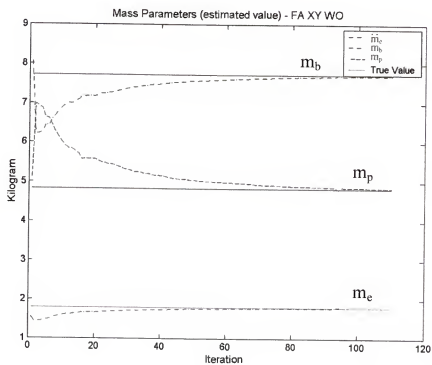


Figure 4.36 Mass Value (XY-direction, Random)

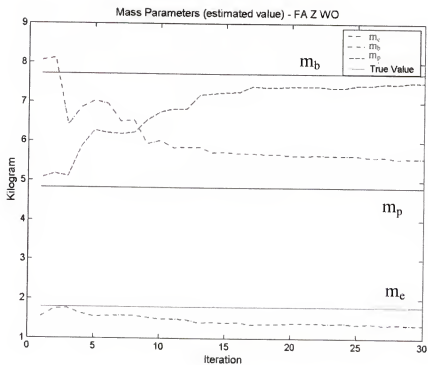


Figure 4.37 Mass Value (Z-direction, Random)

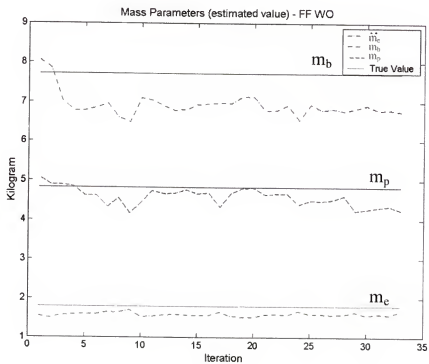


Figure 4.38 Mass Value (FF case, Random)

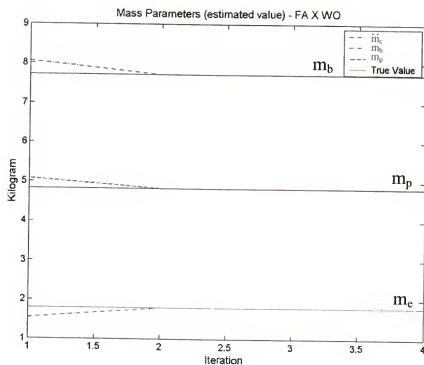


Figure 4.39 Mass Value (X-direction, Spread)

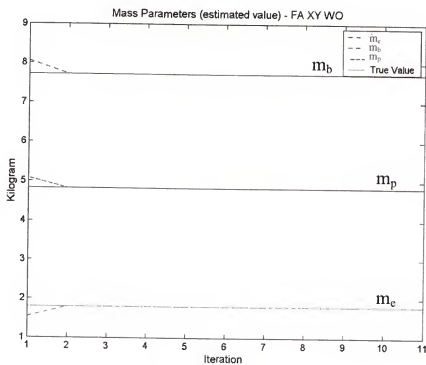


Figure 4.40 Mass Value (XY-direction, Spread)

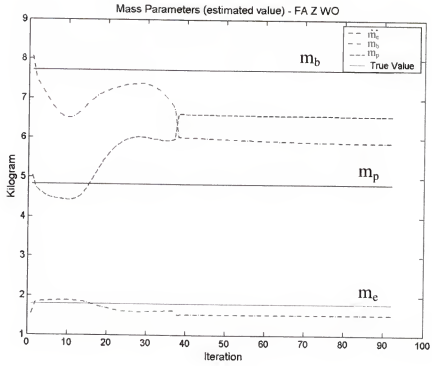


Figure 4.41 Mass Value (Z-direction, Spread)

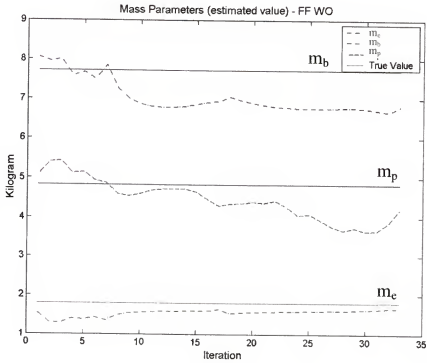


Figure 4.42 Mass Value (FF case, Spread)

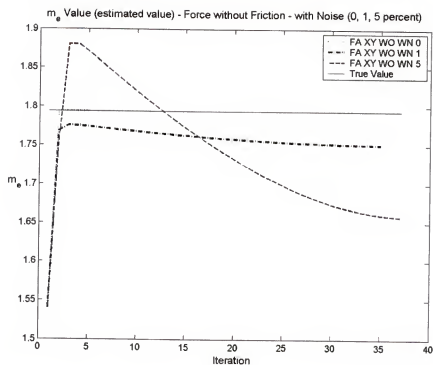


Figure 4.43 m_e Value (0, 1 and 5 % Noise, Force-Based Model)

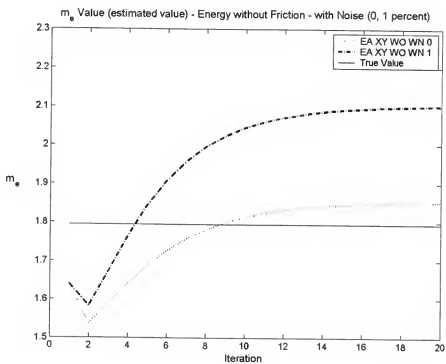


Figure 4.44 m_e Value (0 and 1 % Noise, Energy-Based Model)

CHAPTER 5

PKM EXPERIMENT RESULTS

This chapter presents the experimental setups and the estimated inertial parameter results for the UF-PKM. Since the energy-base method did not perform as well as expected, only the force-based UKF approach is used in this dissertation for the experimental validation study. An ultimate goal for this dissertation is to determine how well the estimation program performs in a real PKM application with machine and instrumentation limitations. Since the strut lengths and individual strut forces are the most readily available data that can be acquired from the machine controller and sensors, the accuracy of this data is a critical for extracting other system states such as strut velocities and accelerations. Shamblin [37] noted that the encoder reading for acquiring strut length data and current reading for acquiring the strut force contained significant noise resulting in measurement inaccuracies. This noise effect was further amplified as numerical approximations were used in determining the strut velocities and accelerations. Therefore, to implement the UKF identification procedure presented in Chapters 2 through 4, sources of error and limitations are expected such as those listed below.

- Signal Noise
- Kinematic Errors
- Measurement Errors
- Controller Parameter and Modeling Errors
- Directional Control Problem
- Following Error
- Poor Encoder/ Current Readings
- Limited Sensing and Actuation

Hence, in the experimental implementation of UKF, corresponding uncertainties and reconciliation techniques had to be taken into consideration throughout the analysis and identification of the system parameters. As discussed in reference [47] and chapter 2, the effects of limited sensing were considered in regards to assuming the geometric center of the platform could be assumed the same as the center of mass. Theoretical results indicate that errors in this approximation would result in errors in the resulting parameter estimation. However, due to the simplicity and the full knowledge of the PKM's components properties, the implementation of the UKF parameter identification algorithm was done assuming all centers of masses per reference [48]. Assuming ideal kinematics, the limited sensing issues (e.g., non-direct measurement of platform motion) were circumvented using kinematics and numerical derivatives to determine the platform position, velocity, and acceleration.

5.1 Experimental Setup

The UF PKM (refer to Figures 2.4 (a) and 5.1) is actuated by Aerotech BM200 3-phase brushless motors with encoder feedback. The motors are powered by Aerotech BA20 amplifiers using pulse-width modulation to provide current control. The motion control software is the open-architecture, PC-based Enhanced Machine Controller, developed by the National Institute of Standard and Technology (NIST), and runs under Real-Time Linux on a Pentium II PC. The interface between the amplifiers, motors, and software is a Servo-to-Go motion controller card. At the present level of controller development, only translational motions of the platform are achievable.

In the experiments, the platform was moved in coordinated motion by executing a single G-code command or a series of commands from a script file. The built-in logging feature was used to record strut lengths and motor current values. From the data, actual

platform position for each time step was obtained using an iterative forward kinematics solver [36]. The platform velocity and acceleration were calculated using the PKM kinematics and the strut length, velocities and acceleration. The strut velocities and accelerations were determined numerically applying the 4-point central difference smoothing equations ([37], equations (5.1) and (5.2)) to filtered position data.

$$(\dot{L}_k)_i = \left(\frac{(-L_{k+2} + 8L_{k+1} - 8L_{k-1} + L_{k-2})_i}{12\Delta t} \right) \quad (5.1)$$

$$(\ddot{L}_k)_i = \left(\frac{(-L_{k+2} + 16L_{k+1} - 30L_k + 16L_{k-1} - L_{k-2})_i}{12(\Delta t)^2} \right) \quad (5.2)$$

where $(L_k)_i$ is the absolute strut length for the i^{th} strut at k^{th} iteration step (equation (2.11)) and Δt is the incremental time step.

The motor current readings were converted to the measured F_a using the motor torque constant, and transmission ratio of the ballscrew. The friction forces along the strut axes were estimated using strut velocity data with a Coulomb and viscous friction model. In prior research on the UF-PKM [37], the model's friction coefficients were obtained experimentally through system identification of each individual strut.

5.2 Challenges to Experimental Validation and Reconciliation

Numerous sources of error presented difficulties in experimentally validating the identification of the dynamic parameters. Much of the error was indirectly caused by the limitations of the servo controller. The PKM's controller uses a PID algorithm to generate the control effort from the encoder measured errors. In tuning the system's controller, the derivative term was found necessary for minimizing oscillations that would occur if only P (proportional) or PI (proportional – integral) control was used. The derivative term also amplified the high frequency vibrations that arose from encoder

noise. The result is that the actual machine motion is characterized by small amplitude, high frequency positional oscillations in addition to the smooth commanded motion.

In addition to the undesired oscillations are the 'false' movements caused by finite encoder resolution. As the motor rotates, the position is read as a series of step moves by the encoder. When the true position is between encoder pulses, the position as read by the encoder is seen to oscillate between the two pulses, introducing oscillations that are not actually occurring. Naturally, differentiating position data to obtain velocity and especially, acceleration, amplifies both the real and false oscillations. This noise propagation is such that acceleration caused solely by encoder noise is of similar magnitude to the acceleration of the commanded motion. For this study, an 8th order Butterworth filter was utilized to eliminate positional noise of frequencies greater than 10Hz, as the commanded machine movements were of much lower frequency. This alleviated much of the problem, but noise at less than 10Hz still existed and became indistinguishable from actual motion.

Errors due to noise are also manifested in empirically determined constants such as friction coefficients. During a friction test, although the commanded velocity is constant, the actual velocity and the corresponding control effort are oscillatory. This contributes to random error when measuring the electrical current required to produce a constant strut velocity. Measurement of the motor torque constant (used for converting current measurements into strut forces) was similarly effected.

Another source of error is unmodeled transmission dynamics. For analysis purposes, the transmission of motor rotation to linear strut movement is assumed constant and rigid. The actual machine uses a flexible rubber coupling between the motor shaft

and ballscrew, acting as a spring-damper system. Thus, oscillations occurring in the motor would not be completely transmitted to the platform motion, as is assumed in the simulation. Additionally, the ballscrew itself acts as a rotational spring, though its effect on error is much less than the flexible coupling.

Compounding the previous errors in the friction and torque constants is the relatively small magnitude of measured dynamic forces in relation to friction forces. Thus, to reduce friction within the system, 3-to-1 precision gearboxes were removed from the original UF-PKM setup. Removing the gearbox also had the effect of reducing motor inertia effects (F_{mb}). Furthermore, because of the dominance of the friction with the UF-PKM, these and other unmodeled dynamics effects were assumed negligible in the implementation of the UKF identification method. Due to time limitations, only two sets of single strut friction coefficients were updated with new experimental results (strut 1 and 4). The single strut friction tests were performed using a horizontal single strut configuration. This eliminated the effects from gravity. The resultant estimated coefficients for the friction model using single strut tests are shown on Table 4.1. Using these results compared to reference [37], reference [37]'s estimates were scaled to provide new estimates for the remaining struts, thus accounting for the change in the gear ratios with the new experimental setup.

5.3 Experiment Results (Without Gearbox)

As stated previously, the experimental data for this dissertation is acquired from experiments conducted with the gearbox removed from the PKM system. In the fully assembled UF-PKM experiments, coordinated x , xy , and z -directional movements were used to generate the response and output data sets. The PKM platform's direction and

speed commanded were the same as those used for the simulated analysis as shown Table 4.3, excluding the freefalling test.

5.3.1 Friction Extraction and Estimation Stages

Since the friction coefficients (both viscous and Coulomb) also need to be estimated using the UKF algorithm, the total number of parameters increased to 24 (3 masses, 9 principal inertias and 12 friction coefficients) The

$$\mathbf{w}_k = [\mathbf{w}_k|_{Chapter 4}, Fric(1,1), Fric(1,2), Fric(2,1), Fric(2,2), Fric(3,1), Fric(3,2), Fric(4,1), Fric(4,2), Fric(5,1), Fric(5,2), Fric(6,1), Fric(6,2)] \quad (5.3)$$

where it is assumed that all struts have identical mass and inertia properties. The principal axes of inertia for each strut are defined along the $\{\hat{\mathbf{s}}_1, \hat{\mathbf{s}}_2, \hat{\mathbf{s}}_{23}\}_i$ axes, and the center of mass locations are those estimated in reference [48]. The parameter identification was first executed for the entire set of 24 parameters. Using the following UKF initial estimates to start the estimation process (Reference [37] single strut friction estimates plus within 90% of CAD-based mass and inertia), the parameter estimation did not converge. This led to implementing a three-stage identification approach.

- **Stage 1.** Conducting an estimation of friction followed by inertia parameter estimation. Stage 1 has two steps:
 - Stage 1-A. The initial guess for the friction estimation is made (e.g., the prior strut estimates [37] and/or the author's single strut results). At this stage, the CAD-based inertia parameters were used as the 'true' inertia values. All three trajectories (x, z, and xy-direction) were tested for estimating the friction parameters.

$$\mathbf{w}_k|_{\text{Stage 1-A}} = [\text{Fric}(1,1), \text{Fric}(1,2), \text{Fric}(2,1), \text{Fric}(2,2), \\ \text{Fric}(3,1), \text{Fric}(3,2), \text{Fric}(4,1), \text{Fric}(4,2), \\ \text{Fric}(5,1), \text{Fric}(5,2), \text{Fric}(6,1), \text{Fric}(6,2)]^T \quad (5.4)$$

- Stage 1-B. After obtaining the estimated friction values for each trajectory, the inertia parameters then are estimated with the estimated friction parameters from Stage 1-A used as the ‘true’ friction values. All three trajectories are conducted as well to estimate the inertia parameters.

$$\mathbf{w}_k|_{\text{Stage 1-B}} = [m_e, m_b, m_p, I_{xxb}, I_{yxb}, I_{zxb}, I_{xyp}, I_{yyp}, I_{zyp}, I_{xxe}, I_{yye}, I_{zxe}]^T \quad (5.5)$$

Note, that the estimated friction parameters from FA_X (Stage 1-A) were used as the ‘true’ friction values in estimating the inertia parameters in FA_X. Also, the estimated friction parameters from FA_Z (Stage 1-A) were used as the ‘true’ friction values in estimating the inertia parameters in FA_Z.

- **Stage 2.** Conducting an estimation of friction followed by inertia parameter estimation with using the data given in Stage 1-A. Stage 2 has two steps:
 - Stage 2-A. The initial guess for the friction estimation is the values from Stage 1-A. At this stage, the CAD-based inertia parameters were used for the ‘true’ inertia values. All three trajectories (x, z, and xy-direction) were tested for estimating the friction parameters.

$$\mathbf{w}_k|_{\text{Stage 2-A}} = \mathbf{w}_k|_{\text{Stage 1-A}} \quad (5.6)$$

- Stage 2-B. After obtaining the estimated friction values for each trajectory, the inertia parameters then are estimated with applying the estimated friction parameters from Stage 2-A used as the ‘true’ friction values. All three trajectories are conducted as well to estimate the inertia parameters.

$$\mathbf{w}_k|_{\text{Stage } 2-B} = \mathbf{w}_k|_{\text{Stage } 1-B} \quad (5.7)$$

Note, that the estimated friction parameters from FA_X (Stage 2-A) were used as the friction values to estimate the inertia parameters in FA_X. Also the estimated friction parameters from FA_Z (Stage 2-A) were used as the friction values to estimate the inertia parameters in FA_Z.

- **Stage 3.** Both the friction and inertia parameters were estimated simultaneously. i.e.,

$$\mathbf{w}_k = [\mathbf{w}_k|_{\text{Chapter 4}}, \text{Fric}(1,1), \text{Fric}(1,2), \text{Fric}(2,1), \text{Fric}(2,2), \\ \text{Fric}(3,1), \text{Fric}(3,2), \text{Fric}(4,1), \text{Fric}(4,2), \\ \text{Fric}(5,1), \text{Fric}(5,2), \text{Fric}(6,1), \text{Fric}(6,2)] \quad (5.8)$$

The data used in this stage as the initial estimate is the combined values from Stage 2-A and Stage 2-B. For example, the estimated friction values from Stage 2-A and the estimated inertia values from Stage 2-B in FA_X were combined into one set and used as the initial parameter values to estimate both the friction and the inertia parameters in FA_X, FA_Z, and FA_XY runs for Stage 3.

5.3.2 Experimental Results with Friction Estimates (Stages 1 and 2)

Table 5.1 shows the estimated friction value using the friction estimation program (Stage 1-A). This estimation uses as its initial value those shown in Table 4.1 and the CAD values of reference [48] as the true values. As can be seen, in x-directional movement (denoted by FA_X), the estimated Coulomb friction value ($\text{Fric}(i,2)$, refer to equation (4.1)) was different from the expected single strut estimated value with ~5 to 250 % error. For the estimated viscous friction value ($\text{Fric}(i,1)$, refer to equation (4.1)), strut 1 and 6 coefficients have quite different values from the single strut estimated values with ~54 and 32, respectively, % error considering all trajectory cases. For z-directional

movement (denoted by FA_Z), the estimated Coulomb friction values are different in most struts with ~5 to 210 % error. The viscous frictions for strut 1, 2, 4, and 6 have an error from the expected value with range of ~36 to 85 %. In xy-directional movement (denoted by FA_XY), the viscous values are worse than the previous cases (up to 123 % error). The Coulomb friction differences from the expected values are in the range of ~15 to 300%. Using these newly estimated values, the inertial parameter estimation was performed as Stage 1-B in estimation. The results for Stage 1-B are shown on Table 5.2. The estimated mass parameters have errors ranging from ~2 to 19 % compared to the expected CAD based estimates. The inertial parameters have errors, which range from 0.7 to 13 % except I_{yye} value, which had the same convergence difficulty as in simulation (Chapter 4).

Since the percent difference in the friction estimations (Stage 1-A) compared to the values in Table 4.1 is too large and the difference between each trajectory values are also too large, the next stage (Stage 2) of estimation is required for identifying which friction parameter values are more reliable. The results are shown on Tables 5.3 and 5.4. Note, that for x-directional movement; the previously estimated friction parameters of Stage 1-A from the x-directional movement case were used as the expected true values for the friction estimation, and similarly for z and xy-directional movements. The estimated friction results for x-directional movement now have an error range of 0 to 4 % from previously estimated friction values. For z-directional movement, the error range for friction was 15 %. Generally, an error less than 2.6% was found for xy motion with exception of one coefficient having ~23% error. With using the resultant friction values from Stage 2-A (Table 5.3), inertia parameters are estimated (Stage 2-B, Table 5.4).

Again, the CAD-based values are assumed the ‘true’ values as basis for comparison of estimated results. In addition, the Stage 1-B values were compared with the estimated values to determine if any convergence to the new estimates is occurring. For mass and inertia parameter values, the error ranges from the CAD values are relatively close to the previous estimates in Table 5.2 (within 13% to 18%). Comparing Stage 1-B estimated values with Stage 2-B estimates indicates that there is possible convergence starting to be reached for each trajectory direction, refer to (% diff Stage 1-B) values in Table 5.4.

5.3.3 Experimental Results with Different Initial Values (Stage 3)

While the friction estimates are converging to values in each respective direction, there does not appear to be a convergence to a single set of parameter values for friction regardless of motion trajectory. Hence, Stage 3 is a search for a single set of friction parameter values using full parameter estimation. This involves using each of the estimates in friction of Stage 2-A and the estimated inertia values from Stage 2-B for each trajectory as the initial values for the other trajectories in Stage 3 (e.g., the use values from FA_X as the initial value for the FA_XY trajectory estimation). The rationale for this is one to avoid estimates to have reached a local minimum as well as to determine an estimate of a single set of parameters. If the program falls into a local minimum inside the UKF, then the convergence could be different from what it is supposed to be. The results using x-directional value are shown on Tables 5.5 and 5.6. Use of other directional movement (xy and z) estimates as initial values were found to not yield convergence in the estimation, thus, the corresponding initial values (estimates of Stage 2-A and Stage 2-B) are not estimates of the ‘true’ values. The observation of the results in x-directional movement estimation case is that the viscous friction values all converged within a 0 to 22 % range, but the Coulomb friction value went up to 0 to 186

% for the FA_XY trajectory. The inertial parameters converged within 5 to 26 % from the CAD values. However, for FA_X and FA_Z trajectory case, when comparing to the Stage 2-B value, the inertia parameters converged within 5 %. For the FA_XY trajectory case, the inertia parameters converged within 39 %. Figure 5.2 thru 5.16 show the convergence tendency for mass parameters and friction parameters. Most of figures have similar tendencies. The xy-directional movement values converged to much higher values than the z-directional movement values, where the x-directional movement values converge to the expected values (estimates from Stage 2 in x-direction). The z-directional movement values converge much closer to the expected values than the xy-directional movement case. For Figure 5.12, it can be seen that although the intermediate estimates diverged from expected at some stage of iteration, its value returned quickly to an estimate, converging to within -8.67 % of expected.

5.4 Summary of Results

For ‘without gearbox’ case, several observations are made for estimating the parameters. For friction case, the direct measurement from the individual struts using the single strut tests does not capture the spherical and Hooke joint friction values. This can be delineated from the estimation results for the different input trajectories. E.g., the X- and XY- motions move the struts over different joint ranges while the Z-motion yields joint motion closer to the X-motion case. Selecting as initial values the estimates of prior estimations determined along different motion trajectories quickly narrowed the possible results towards finding the true values of the system. The inertia parameters exhibited a much faster convergence to a ‘true’ value. To conclude, the single strut estimates have not fully captured the strut friction. Thus, the new friction estimates are considered more accurate. The inertia estimates showed very good, accurate convergence especially when

considering the fact that friction forces dominated the data. However, the differences between friction estimates per the different trajectories still needs further resolve as to the determination of the true rotational friction differences of each spherical and Hooke joint's value. Hence, the accurate modeling for spherical and Hooke joints are highly recommended for better explanation of these possible phenomena and has been left for future work.

To conclude, the recommended identification methodology for actual implementation on PKM's is the multistage iterative approach presented in section 5.3.1 using the force-based UKF model algorithm of Chapter 2. This is particularly true for systems with a large amount of friction. The following highlights some of the critical issues.

- First, one should not ignore the importance of the initial guess for the friction parameters (the usual dominance found in most ballscrew driven machines). Thus, the tests with different trajectories (i.e. x, z, and xy-directional movement) must be conducted for finding the possible friction parameter values. (Stages 1 and 2)
- Since the estimation program could fall into the local minimum for the highly nonlinear systems, the estimated friction parameter values obtained from the previous step must be tested again with other trajectories. (e.g. the estimated friction parameters from x-directional movement test must be performed with xy- and z-directional movement test.

- The final friction parameters, that are found to show estimateion convergence in all other trajectories, should be taken into consideration as the closer value to the true value.

Table 5.1 Friction Estimation with CAD Based Inertia Assumed True Values (Stage 1-A)

Parameters	Expected Value (Single Strut)	FA_X % from Expected Value	FA_Z % from Expected Value	FA_XY % from Expected Value
Fric(1,1)	1258.00	-53.42	2327.04	-84.96
Fric(1,2)	50.00	-251.15	154.34	-208.63
Fric(2,1)	1012.00	-3.54	1387.72	-36.81
Fric(2,2)	49.00	-23.61	70.36	-42.81
Fric(3,1)	1099.00	-16.24	1276.86	-16.09
Fric(3,2)	58.00	4.93	61.56	-5.86
Fric(4,1)	3597.17	-2.28	4915.50	-39.93
Fric(4,2)	131.00	-114.34	227.82	-74.21
Fric(5,1)	1140.00	-14.13	1315.01	-15.47
Fric(5,2)	49.00	-33.55	65.80280	-34.57
Fric(6,1)	1188.00	-31.62	1646.49	-40.79
Fric(6,2)	53.00	-99.18	93.47	-75.95
			194.18	-265.94

(Units – Fric (i,1)=Ns/m, Fric (i,2)=N, i=1,...,6)

Table 5.2 Estimated Inertia Values Using Estimated Friction from Stage 1-A Data (Stage 1-B)

Parameters	True Value	FA_X	FA_X % Diff	FA_Z	FA_Z % Diff	FA_XY	FA_XY % Diff
m_c	1.79	1.65	8.31	1.60	10.49	1.46	18.81
m_b	7.72	8.73	-13.31	8.33	-7.99	7.95	-2.91
m_p	4.82	5.43	-12.78	5.24	-8.76	4.95	-2.53

(Units – kilogram)

Parameters	True Value	FA_X	FA_X % Diff	FA_Z	FA_Z % Diff	FA_XY	FA_XY % Diff
Ixxb	563500.00	511441.00	9.24	515543.40	8.51	516349.50	8.37
Iyyb	69400.00	60110.00	13.39	60481.30	12.85	60554.88	12.75
Izzb	563500.00	500468.00	11.19	503579.70	10.63	504408.80	10.49
Ixxp	90761.00	89438.51	1.46	89990.37	0.85	90099.85	0.73
Iyyp	181283.60	178815.40	1.36	179918.80	0.75	180137.70	0.63
Izzp	90761.21	89422.62	1.48	89974.37	0.87	90083.84	0.75
Ixxe	18269.40	17918.09	1.92	18029.67	1.31	18051.80	1.19
Iyye	54.19	39.73	26.68	39.98	26.23	40.03	26.14
Izze	18269.40	17924.01	1.89	18034.63	1.29	18056.80	1.16

(all times 1×10^{-6} , Units – Nm^2)

Table 5.3 Estimated Friction Value Using Previously Estimated Friction Value from Stage 1-A as Initial Value (Stage 2-A)

Parameters	Expected Value X (Stage 1-A estimates for X)	FA_X	FA_X % from Expected Value	Expected Value Z (Stage 1-A estimates for Z)	FA_Z	FA_Z % from Expected Value	Expected Value XY (Stage 1-A estimates for XY)	FA_XY	FA_XY % from Expected Value
Fric(1,1)	1933.79	1933.91	-0.01	2327.04	2285.26	1.77	2809.01	2827.28	-0.67
Fric(1,2)	176.11	176.34	-0.14	154.34	145.84	5.42	195.25	200.15	-2.59
Fric(2,1)	1046.88	1048.67	-0.14	1387.72	1397.43	-0.85	1352.02	1353.96	-0.14
Fric(2,2)	60.43	60.56	-0.08	70.36	72.08	-2.84	105.20	104.14	1.06
Fric(3,1)	1277.97	1278.76	-0.08	1276.86	1331.09	-4.14	1712.84	1711.02	0.13
Fric(3,2)	55.22	55.39	-0.36	61.56	68.52	-11.02	164.12	161.78	1.60
Fric(4,1)	3597.17	3570.85	0.73	4915.50	4576.46	6.99	6042.66	6059.21	-0.27
Fric(4,2)	283.30	288.33	-1.98	227.82	192.26	15.82	414.00	424.43	-2.51
Fric(5,1)	1301.25	1303.43	-0.18	1315.01	1256.75	4.45	1307.75	1320.11	-0.98
Fric(5,2)	65.47	62.99	3.71	65.80	58.40	11.30	113.38	138.43	-22.71
Fric(6,1)	1541.17	1540.27	0.03	1646.49	1606.44	2.37	2336.60	2348.51	-0.53
Fric(6,2)	106.16	106.21	-0.18	93.47	86.85	6.88	194.18	198.31	-2.21

(Units – Fric (i,1)=Ns/m, Fric (i,2)=N, i=1,...,6)

Table 5.4 Estimated Inertial Value Using Previously Estimated Friction Value from Stage 2-A (Stage 2-B)

Parameters	True Value	FA_X (Stage 1-B)	FA_X % Diff (% Diff Stage 1-B)	FA_Z (Stage 1-B)	FA_Z % Diff (% Diff Stage 1-B)	FA_XY (Stage 1-B)	FA_XY % Diff (% Diff Stage 1-B)
m_c	1.79	1.56 (1.65)	12.92 (5.45)	1.60 (1.60)	11.00 (0.00)	1.46 (1.46)	18.52 (0.00)
m_b	7.72	8.08 (8.73)	-4.71 (7.45)	8.11 (8.33)	-4.99 (2.64)	7.96 (7.95)	-3.08 (-0.13)
m_p	4.82	5.11 (5.43)	-6.11 (5.89)	5.16 (5.24)	-7.06 (1.53)	4.96 (4.95)	-2.75 (-0.20)

(Units – kilogram) Note, (Stage 1-B) stands for the values at Stage 1-B and (%Stage 1-B) represents the % differences between the estimated parameter values at Stage 2-B and the estimated parameter values at Stage 1-B.

Parameters	True Value	FA_X (Stage 1-B)	FA_X % Diff (% Diff Stage 1-B)	FA_Z (Stage 1-B)	FA_Z % Diff (% Diff Stage 1-B)	FA_XY (Stage 1-B)	FA_XY % Diff (% Diff Stage 1-B)
Ixxb	563500.00	515955.90 (511411.00)	8.44 (-0.88)	516126.90 (515543.40)	8.41 (-0.11)	515788.00 (516349.50)	8.47 (0.11)
Iyyb	69400.00	60510.56 (60110.00)	12.81 (-0.67)	60522.07 (60481.30)	12.79 (-0.07)	60489.64 (60554.88)	12.84 (0.11)
Izzb	563500.00	504032.90 (500468.00)	10.55 (-0.71)	504158.60 (503579.70)	10.53 (-0.11)	503865.30 (504408.80)	10.58 (0.11)
Ixxp	90761.00	90033.91 (89438.51)	0.80 (-0.67)	90051.04 (89990.37)	0.78 (-0.07)	90002.78 (90099.85)	0.84 (0.11)
Iyyp	181283.60	180005.80 (178815.40)	0.71 (-0.67)	180040.10 (179918.80)	0.69 (-0.07)	179943.60 (180137.70)	0.74 (0.11)
Izzp	90761.21	90017.91 (89422.62)	0.82 (-0.67)	90035.03 (89974.37)	0.80 (-0.07)	89986.79 (90083.84)	0.8 (0.11)
Ixxc	18269.40	18038.57 (17918.09)	1.26 (-0.67)	18042.08 (18029.67)	1.24 (-0.07)	18032.35 (18051.80)	1.20 (0.11)
Iyye	54.19	40.00 (39.73)	26.20 (-0.68)	40.00 (39.98)	26.18 (-0.05)	40.00 (40.03)	26.22 (0.07)
Izze	18269.40	18043.58 (17924.01)	1.24 (-0.67)	18047.04 (18034.63)	1.22 (-0.07)	18037.35 (18056.80)	1.27 (0.11)

(all times 1×10^{-6} , Units – Nm²)

Table 5.5 Estimated Friction Value Using Previously Estimated Parameters as Initial Value (Stage 3)

Parameters	Expected Value	FA_X % from Expected Value	FA_Z	FA_Z % from Expected Value	FA_XY	FA_XY % from Expected Value
Fric(1,1)	1933.91	-4.59	1888.33	2.33	2040.42	-5.48
Fric(1,2)	176.34	-0.84	155.01	11.86	215.94	-22.45
Fric(2,1)	1048.67	3.73	1156.84	-10.82	1168.36	-11.68
Fric(2,2)	60.56	-2.00	79.94	-33.23	99.40	-65.14
Fric(3,1)	1278.76	0.17	1403.67	-9.70	1897.87	-48.72
Fric(3,2)	55.39	-2.44	67.37	-21.68	157.80	-186.59
Fric(4,1)	3570.85	0.03	3385.07	5.92	3927.05	-9.13
Fric(4,2)	288.33	-8.67	210.50	25.80	450.07	-58.62
Fric(5,1)	1303.43	-5.02	1259.36	3.17	1450.81	-11.44
Fric(5,2)	62.99	2.29	58.85	9.96	151.57	-130.89
Fric(6,1)	1540.27	-4.00	1482.06	3.78	1881.00	-22.05
Fric(6,2)	106.21	-3.08	91.78	13.33	211.50	-99.22

(Units – Fric (i,1)=Ns/m, Fric (i,2)=N, i=1,...,6)

Previously Estimated Parameters: x-directional movement, from Tables 5.3 and 5.4 (Stages 2-A and 2-B)

Table 5.6 Estimated Inertial Value Using Previously Estimated Value (x-directional movement from Tables 5.3 and 5.4 (Stages 2-A and 2-B)) as Initial Value (Stage 3)

Parameters	CAD Value (Stage 2-B)	FA_X	FA_X % Diff (% Diff Stage 2-B)	FA_Z	FA_Z % Diff (% Diff Stage 2-B)	FA_XY	FA_XY % Diff (% Diff Stage 2-B)
m _c	1.79 (1.56)	1.63	9.37 (-4.49)	1.58	11.86 (-1.28)	2.17	-20.76 (-39.10)
m _b	7.72 (8.08)	8.17	-5.72 (-1.11)	8.09	-4.94 (-0.12)	8.90	-15.37 (-10.15)
m _p	4.82 (5.11)	5.22	-8.15 (-2.15)	5.13	-6.67 (-0.39)	6.09	-26.40 (-19.18)

(Units – kilogram) Note, (Stage 2-B) stands for the values at Stage 2-B (FA_X) and (%Stage 2-B) represents the % differences between the estimated parameter values at Stage 3 and the estimated parameter values at Stage 2-B (FA_X).

Parameters	CAD Value (Stage 2-B FA_X)	FA_X	FA_X % Diff (% Diff Stage 2-B)	FA_Z	FA_Z % Diff (% Diff Stage 2-B)	FA_XY	FA_XY % Diff (% Diff Stage 2-B)
I _{xxb}	563500.00 (515955.90)	515951.70	8.44 (0.00)	516012.90	8.43 (-0.01)	515150.00	8.58 (0.16)
I _{yyb}	69400.00 (60510.56)	60515.99	12.80 (-0.01)	60516.00	12.80 (-0.01)	60411.74	12.95 (0.16)
I _{zzb}	563500.00 (504032.90)	504072.10	10.55 (-0.01)	504079.80	10.55 (-0.01)	503560.30	10.64 (0.09)
I _{xxp}	90761.00 (90033.91)	90041.99	0.79 (-0.01)	90042.00	0.79 (-0.01)	89886.87	0.96 (0.16)
I _{yy p}	181283.60 (180005.80)	180022.00	0.70 (-0.01)	180022.00	0.70 (-0.01)	179711.90	0.87 (0.16)
I _{zzp}	90761.21 (90017.91)	90025.99	0.81 (-0.01)	90026.00	0.81 (-0.01)	89870.90	0.98 (0.16)
I _{xxe}	18269.40 (18038.57)	18040.10	1.26 (-0.01)	18040.20	1.26 (-0.01)	18009.15	1.43 (0.16)
I _{yye}	54.19 (40.00)	40.00	26.19 (0.00)	40.00	26.19 (0.00)	39.93	26.32 (0.17)
I _{zze}	18269.40 (18043.58)	18045.19	1.23 (-0.01)	18045.20	1.237 (-0.01)	18014.48	1.40 (0.16)

(all times 1×10^{-6} , Units – Nm²)

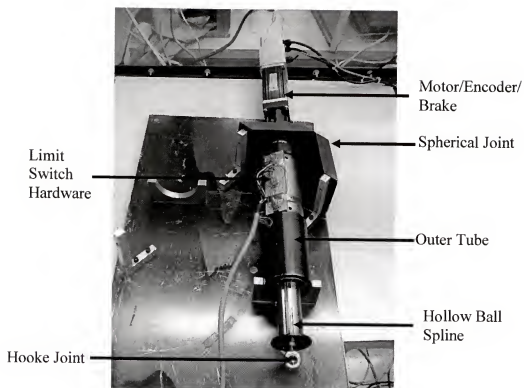


Figure 5.1 Single Strut Testing

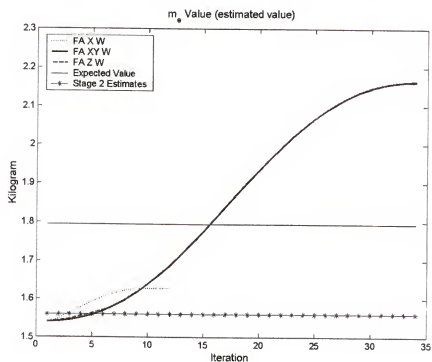
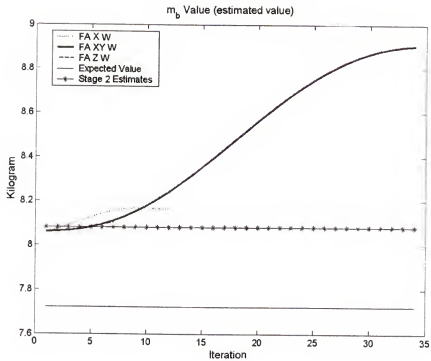
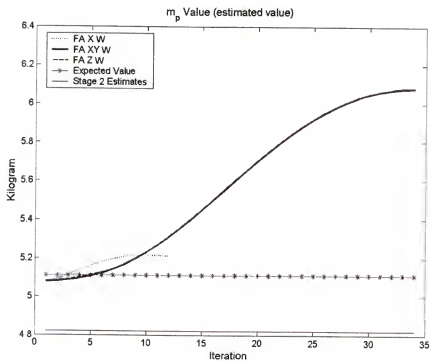


Figure 5.2 Value of m_e (Experimental Result)

Figure 5.3 Value of m_b (Experimental Result)Figure 5.4 Value of m_p (Experimental Result)

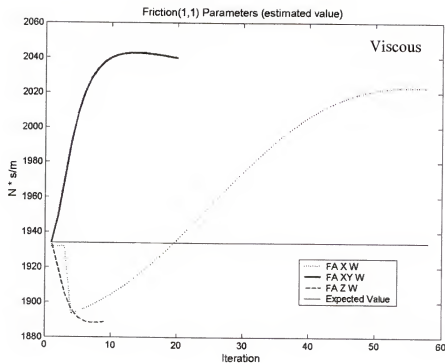


Figure 5.5 Friction (1,1) Parameter Value (Experimental Result)

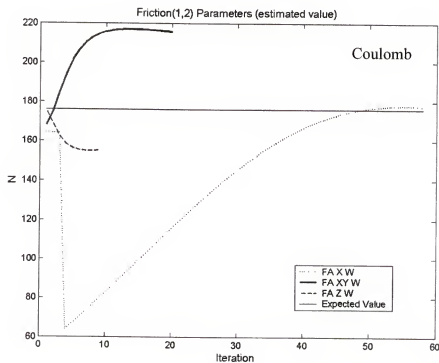


Figure 5.6 Friction (1,2) Parameter Value (Experimental Result)

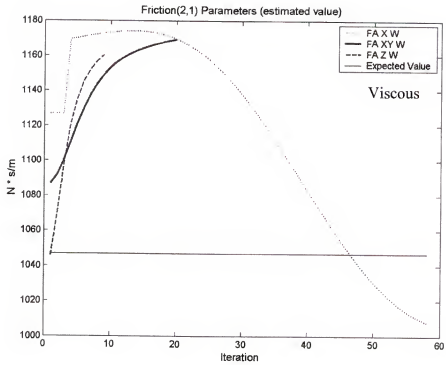


Figure 5.7 Friction (2,1) Parameter Value (Experimental Result)

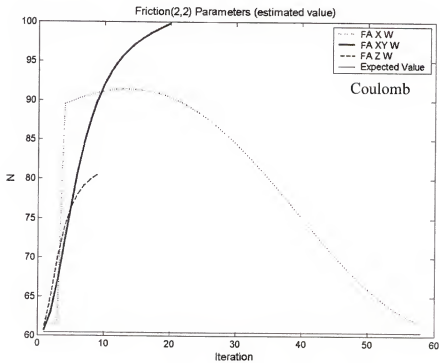


Figure 5.8 Friction (2,2) Parameter Value (Experimental Result)

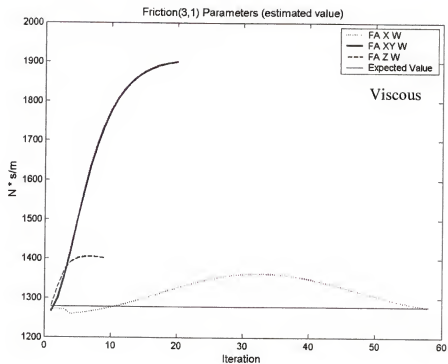


Figure 5.9 Friction (3,1) Parameter Value (Experimental Result)

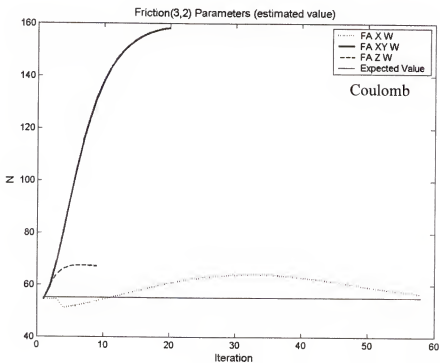


Figure 5.10 Friction (3,2) Parameter Value (Experimental Result)

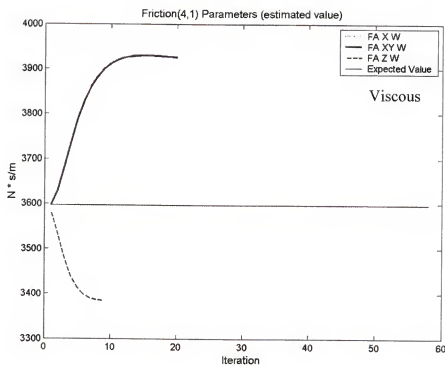


Figure 5.11 Friction (4,1) Parameter Value (Experimental Result)

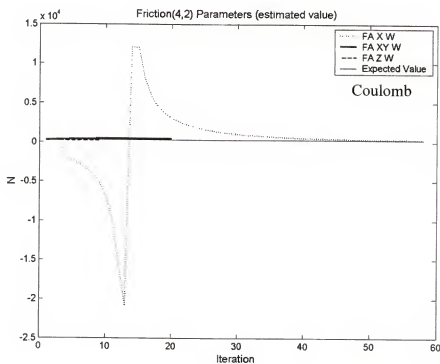


Figure 5.12 Friction (4,2) Parameter Value (Experimental Result)

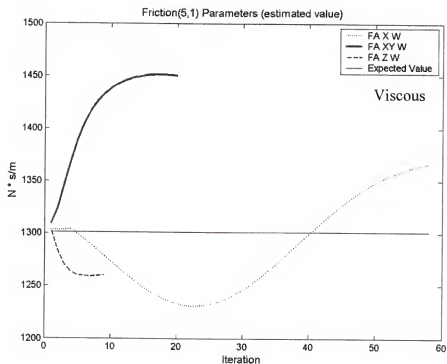


Figure 5.13 Friction (5,1) Parameter Value (Experimental Result)

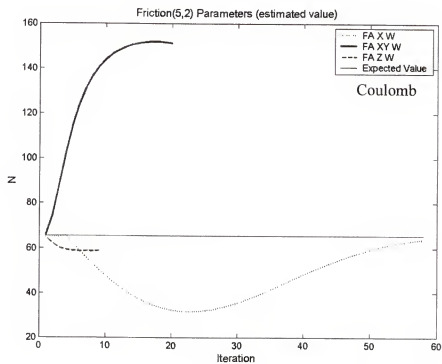


Figure 5.14 Friction (5,2) Parameter Value (Experimental Result)

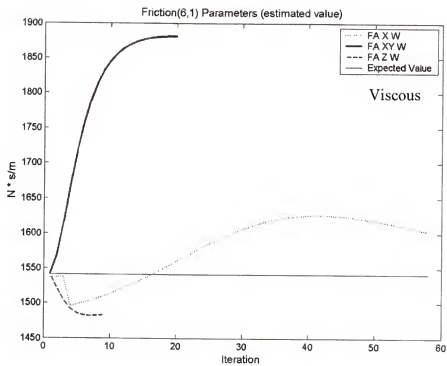


Figure 5.15 Friction (6,1) Parameter Value (Experimental Result)

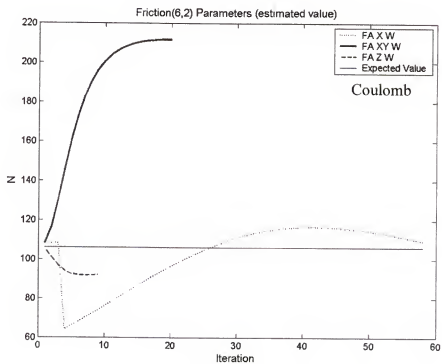


Figure 5.16 Friction (6,2) Parameter Value (Experimental Result)

CHAPTER 6 CONCLUSION

To date, very little research on the dynamics of parallel kinematic mechanisms (PKMs) exists. Of the work available in the literature, most have been for generating the equations of motion, inverse dynamics, and structural dynamics. Insight to the structure and properties of PKM dynamics, effects of strut inertias, dynamics interactions with various kinematics error sources, formulation of dynamic performance indices, and dynamic balancing for PKMs have only recently been provided. To address the increasing awareness of the dynamics needs, which are brought to the forefront by the above research or lack thereof, the research of this dissertation established a dynamic parameter identification approach for PKMs. Thus, this enables engineers to easily acquire the values of the parameters necessary for obtaining a valid dynamic model of the system.

To choose the method used in this research, several identification methods were considered. The main comparisons were made between least squares method, extended Kalman filter and unscented Kalman filter. Using the inverted double pendulum as a sample test system in the comparison study, it was found that the UKF performance was overall superior to the others. Thus, UKF was selected and implemented for parameter identification of PKM and tested via simulation study and validated with experimental implementation on the UF PKM.

The simulations studies for the UF Special 6-6 PKM had two different categories: with friction and without friction. Prior work indicated a high degree of friction within

the UF-PKM system. The trajectories were chosen to exhibit x , z , and xy direction relative to the inertia frame. Other comparisons were made involving different tests for generating response data, e.g., actively controlled strut test and ‘freefalling’ test.

The outcomes of the simulations were that the estimated mass values generally converged well within 1 percent and for inertia (excluding I_{yye}) values converged within 1 to 12 percent for systems ‘without-friction’ test. When the values of the inertia were very small compared to other parameter values (e.g., I_{yye}), the convergence percentage of the inertia was only within 26 percent. For ‘with-friction’ test, the convergence trend was similar to the ‘without-friction’ test. The other results show that ‘freefalling’ test did not yield improved identification over the use of the ‘actively controlled strut’ test. This lack of expected improvement is in part attributed to the limitation of UF Special 6-6 PKM range of motion as well as the geometric effects of struts approaching a parallel configuration. The ‘freefalling’ test needs more data if it is to be used with this methodology is to be used. In addition, force-based and energy-based model methods were compared to determine if there is any benefit other than reduced model effort in deriving the theoretical model. A significant improvement using energy-based method was not generally realized. As an additional comparison between the force-based model and the energy-based model cases without friction, noise was added into the UKF estimation. The mass parameters are estimated within 9.2 % for force-based model and 80.5 % for energy-based model when the 1% noise is added. This result shows that the force-based model is the highly preferred model-base to be used for the UKF.

The experimental study for the UF Special 6-6 PKM had three stages to be conducted. In Stage 1, separate parameter sets were estimated sequentially (i.e., friction

parameter estimation followed by inertia parameter estimation using initial friction parameter CAD based inertia estimates). In Stage 2, again involves separate parameter sets sequentially estimated using Stage 1 friction parameter estimates as initial values for the UKF. Stages 1 and 2 are compared for determining potential convergence. In Stage 3, the results of Stage 2 are used as initial values in UKF for full parameter estimation (both friction and inertia parameter). The initial parameters using each directional estimates of Stage 2 were found either to converge or not. Thus, the outcome allows quickly narrows the possible results by eliminating the non-convergence case's initial values as possible estimates of the 'true' value. The experimental implementation's inertial parameter estimation was considered to exhibit high degree accuracy, especially in the presence of high friction conditions and a large degree of uncertainties in kinematic parameter.

In conclusion, the development of an enhanced dynamic identification methodology for PKM was established and implemented. The simulated results show the unscented Kalman filter successfully applied for identifying the system parameters of PKMs. Since the unscented Kalman filter uses the original nonlinear equations, the procedure for obtaining a linearized model is not necessary. This merit corresponds to the significant reduction in modeling effort required to generate the basic algorithm used for the system identification. In an industrial setting, since a simple method to be used would be required, the unscented Kalman filter would be the best choice. The recommended identification methodology for actual implementation on PKM is the multistage iterative approach using the force-based UKF model. This is particularly true for systems with a large amount friction. The highlights of some of the critical issues

were presented in the summary of Chapter 5. In brief they were various trajectories must be utilized so as to obtain better estimates of parameters especially in the presence of friction and to avoid estimation process falling into local minimum.

The overall impact to technology is provision of a parameter identification method that yields improved rigid body dynamics models of parallel kinematic mechanisms. This will lead to the ability to introduce advanced controllers that exploit the model dynamics to improve system performance. In addition, with some refinement, the presented method sets the foundation for formulating an online tuning algorithm.

Other future variants to the research worthy of exploration is:

- Additional testing to include identification of the center of mass position in the UKF experimental implementation.
- Dual estimation testing (parameter and state estimation).
- Use of alternate strut force sensing (e.g., strain gage testing).
- On-line adaptive tuning.
- More rigorous approach for generating dynamically exciting trajectories.

APPENDIX A THE Z-X-Z EULER ANGLE DEFINITION AND [U] MATRIX FORMATION

The z-x-z Euler angles are defined as the ordered sequence of rotations of coordinate systems with rotations about z-axis, rotated x-axis, and rotated z-axis. The procedure of the z-x-z Euler angle rotations are as follows (as shown in Figure A.1).

1. The base coordinate system ($\hat{\mathbf{e}}$ coordinate system, same as $\{F\}$ coordinate system in Chapter 2) is rotated to $\hat{\mathbf{f}}$ coordinate system with angle ϕ with respect to z-axis ($\hat{\mathbf{e}}_3$). (refer to Figure A.1 (1))
2. Next procedure is that the $\hat{\mathbf{f}}$ coordinate system is rotating to the $\hat{\mathbf{g}}$ coordinate system with angle θ with respect to x-axis ($\hat{\mathbf{f}}_1$). (refer to Figure A.1 (2))
3. Final procedure is that the $\hat{\mathbf{g}}$ coordinate system is rotating to the $\hat{\mathbf{b}}$ coordinate system with angle ψ with respect to z-axis ($\hat{\mathbf{g}}_3$). (refer to Figure A.1 (3)) The $\hat{\mathbf{b}}$ coordinate system is same as $\{P\}$ coordinate system in Chapter 2.

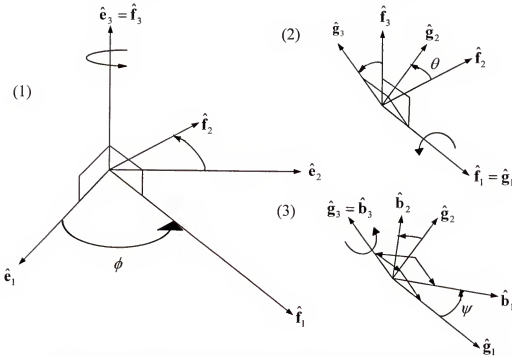


Figure A.1 The z-x-z Euler Angle Expression in the Coordinate Systems

Figure A.1 can be regenerated with the individual coordinate systems as shown in figure A.2.

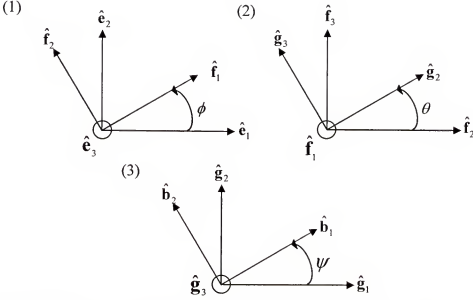


Figure A.2 The z-x-z Euler Angle Expression in Individual Coordinate Systems

The corresponding expressions for the rotational matrices for the individual coordinate systems are:

$$\underline{\hat{\mathbf{f}}} = \begin{bmatrix} c\phi & s\phi & 0 \\ -s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{\hat{\mathbf{e}}} = [l_3(\phi)] \underline{\hat{\mathbf{e}}} \quad (\text{A.1})$$

$$\underline{\hat{\mathbf{g}}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & s\theta \\ 0 & -s\theta & c\theta \end{bmatrix} \underline{\hat{\mathbf{f}}} = [l_1(\theta)] \underline{\hat{\mathbf{f}}} \quad (\text{A.2})$$

$$\underline{\hat{\mathbf{b}}} = \begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{\hat{\mathbf{g}}} = [l_2(\psi)] \underline{\hat{\mathbf{g}}} \quad (\text{A.3})$$

$$\hat{\mathbf{b}} = [l_3(\psi)] [l_1(\theta)] [l_3(\phi)] \hat{\mathbf{e}} \quad (\text{A.4 (a)})$$

$$\hat{\mathbf{b}} = \begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & s\theta \\ 0 & -s\theta & c\theta \end{bmatrix} \begin{bmatrix} c\phi & s\phi & 0 \\ -s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \hat{\mathbf{e}} \quad (\text{A.4 (b)})$$

$$\hat{\mathbf{b}} = \begin{bmatrix} c\psi & s\psi c\theta & s\psi s\theta \\ -s\psi & c\psi c\theta & c\psi s\theta \\ 0 & -s\theta & c\theta \end{bmatrix} \begin{bmatrix} c\phi & s\phi & 0 \\ -s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \hat{\mathbf{e}} \quad (\text{A.4 (c)})$$

$$\hat{\mathbf{b}} = \begin{bmatrix} c\psi c\phi - s\psi c\theta s\phi & c\psi s\phi + s\psi c\theta c\phi & s\psi s\theta \\ -s\psi c\phi - c\psi c\theta s\phi & -s\psi s\phi + c\psi c\theta c\phi & c\psi s\theta \\ s\theta s\phi & -s\theta c\phi & c\theta \end{bmatrix} \hat{\mathbf{e}} \quad (\text{A.4 (d)})$$

$$\hat{\mathbf{b}} = {}^{\hat{\mathbf{b}}} \mathbf{R}_{\hat{\mathbf{e}}} \hat{\mathbf{e}} \quad (\text{A.4 (e)})$$

and

$$\hat{\mathbf{e}} = {}^{\hat{\mathbf{b}}} \mathbf{R}_{\hat{\mathbf{e}}}^T \hat{\mathbf{b}} \quad (\text{A.5})$$

and

$$\text{also } {}^{\hat{\mathbf{e}}} \mathbf{R}_{\hat{\mathbf{b}}} = {}^{\hat{\mathbf{b}}} \mathbf{R}_{\hat{\mathbf{e}}}^T \quad (\text{A.6})$$

where

$\hat{\mathbf{e}}$ the inertial coordinate

$\hat{\mathbf{f}}$ the first intermediate rotational coordinate of first Euler angle ϕ

$\hat{\mathbf{g}}$ the second intermediate rotational coordinate of second Euler angle θ

$\hat{\mathbf{b}}$ the third intermediate rotational coordinate of third Euler angle ψ

${}^{\hat{\mathbf{b}}} \mathbf{R}_{\hat{\mathbf{e}}}$ the rotational matrix from the $\hat{\mathbf{e}}$ coordinate system to the $\hat{\mathbf{b}}$ coordinate system

${}^{\hat{\mathbf{e}}}\mathbf{R}_{\hat{\mathbf{b}}}$ the rotational matrix from the $\hat{\mathbf{b}}$ coordinate system to the $\hat{\mathbf{e}}$ coordinate system

For finding [U] matrix, the angular velocity of $\hat{\mathbf{b}}$ coordinate systems relative to the $\hat{\mathbf{e}}$ coordinate systems can be expressed as simply adding the three intermediate angular velocities expressed in the same coordinate system.

$${}^{\hat{\mathbf{e}}}\boldsymbol{\omega}^{\hat{\mathbf{b}}} = {}^{\hat{\mathbf{e}}}\boldsymbol{\omega}^{\hat{\mathbf{f}}} + {}^{\hat{\mathbf{f}}}\boldsymbol{\omega}^{\hat{\mathbf{g}}} + {}^{\hat{\mathbf{g}}}\boldsymbol{\omega}^{\hat{\mathbf{b}}} = {}^{\hat{\mathbf{e}}}\boldsymbol{\omega}^{\hat{\mathbf{f}}} + [L_3(\phi)]^T {}^{\hat{\mathbf{f}}}\boldsymbol{\omega}^{\hat{\mathbf{g}}} + [L_3(\phi)]^T [L_1(\theta)]^T {}^{\hat{\mathbf{g}}}\boldsymbol{\omega}^{\hat{\mathbf{b}}} \quad (\text{A.7 (a)})$$

$${}^{\hat{\mathbf{e}}}\boldsymbol{\omega}^{\hat{\mathbf{b}}} = \dot{\phi} \hat{\mathbf{e}}_3 + \dot{\theta} \hat{\mathbf{f}}_1 + \dot{\psi} \hat{\mathbf{g}}_3 \quad (\text{A.7 (b)})$$

$${}^{\hat{\mathbf{e}}}\boldsymbol{\omega}^{\hat{\mathbf{b}}} = \dot{\phi} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}_{\hat{\mathbf{e}}} + \dot{\theta} [L_1(\phi)]^T \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}_{\hat{\mathbf{f}}} + \dot{\psi} [L_3(\phi)]^T [L_1(\theta)]^T \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}_{\hat{\mathbf{g}}} \quad (\text{A.7 (c)})$$

where ${}^{\hat{\mathbf{e}}}\boldsymbol{\omega}^{\hat{\mathbf{b}}}$ is the rate of change in the $\hat{\mathbf{b}}$ coordinate system with respect to the $\hat{\mathbf{e}}$ coordinate system. The ${}^{\hat{\mathbf{e}}}\boldsymbol{\omega}^{\hat{\mathbf{f}}}$ is the rate of change in the $\hat{\mathbf{f}}$ coordinate system with respect to the $\hat{\mathbf{e}}$ coordinate system. The ${}^{\hat{\mathbf{f}}}\boldsymbol{\omega}^{\hat{\mathbf{g}}}$ is the rate of change in the $\hat{\mathbf{g}}$ coordinate system with respect to the $\hat{\mathbf{f}}$ coordinate system. The ${}^{\hat{\mathbf{g}}}\boldsymbol{\omega}^{\hat{\mathbf{b}}}$ is the rate of change in the $\hat{\mathbf{b}}$ coordinate system with respect to the $\hat{\mathbf{g}}$ coordinate system. Then

$${}^{\hat{\mathbf{e}}}\boldsymbol{\omega}^{\hat{\mathbf{b}}} = \dot{\phi} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}_{\hat{\mathbf{e}}} + \dot{\theta} \begin{Bmatrix} c\phi \\ s\phi \\ 0 \end{Bmatrix}_{\hat{\mathbf{f}}} + \dot{\psi} \begin{Bmatrix} s\phi s\theta \\ -c\phi s\theta \\ c\theta \end{Bmatrix}_{\hat{\mathbf{g}}} \quad (\text{A.8 (a)})$$

$${}^{\hat{\mathbf{e}}}\boldsymbol{\omega}^{\hat{\mathbf{b}}} = \begin{bmatrix} 0 & c\phi & s\phi s\theta \\ 0 & s\phi & -c\phi s\theta \\ 1 & 0 & c\theta \end{bmatrix} \begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix}_{\hat{\mathbf{e}}} \quad (\text{A.8 (b)})$$

$$\therefore {}^i\omega^{\dot{\mathbf{b}}} = \begin{bmatrix} 0 & c\dot{\phi} & s\dot{\phi}s\dot{\theta} \\ 0 & s\dot{\phi} & -c\dot{\phi}s\dot{\theta} \\ 1 & 0 & c\dot{\theta} \end{bmatrix} \begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = [\mathbf{U}]\dot{\mathbf{\beta}} \quad (\text{A.8 (c)})$$

where ${}^{\dot{\mathbf{e}}}\omega^{\dot{\mathbf{b}}}$ is same as ω_c in Chapter 2.

APPENDIX B
SUPPLEMENTAL DERIVATION DETAILS OF THE EQUATIONS OF MOTION

For equation 2.55

$$\mathbf{F}_a = \mathbf{J}^{-1} \left[\begin{bmatrix} \mathbf{F}_{pm} \\ \mathbf{T}_{pm} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{sm} \\ \mathbf{T}_{sm} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{cor} \\ \mathbf{T}_{cor} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{cen} \\ \mathbf{T}_{cen} \end{bmatrix} \right. \\ \left. + \begin{bmatrix} \mathbf{F}_{grp} \\ \mathbf{T}_{grp} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{gra} \\ \mathbf{T}_{gra} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{frict} \\ \mathbf{T}_{frict} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{ext} \\ \mathbf{T}_{ext} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{m/b} \\ \mathbf{T}_{m/b} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{umd} \\ \mathbf{T}_{umd} \end{bmatrix} \right]$$

The following are the expressions for each of the terms on the right-hand side of the equation. First, for the platform acceleration dependent terms, the forces and

moments are grouped as $\begin{bmatrix} \mathbf{F}_{pm} \\ \mathbf{T}_{pm} \end{bmatrix}$.

$$\begin{bmatrix} \mathbf{F}_{pm} \\ \mathbf{T}_{pm} \end{bmatrix} = [\mathbf{M}_p] \begin{Bmatrix} \mathbf{a}_c \\ \ddot{\mathbf{p}} \end{Bmatrix} \quad (\text{B.1})$$

$$\text{where } [\mathbf{M}_p] = \begin{bmatrix} m_p [\mathbf{I}] & [0] \\ [0] & \begin{bmatrix} \frac{\partial \boldsymbol{\omega}}{\partial \dot{\boldsymbol{\beta}}} \end{bmatrix}^T [\mathbf{I}_p] \begin{bmatrix} \frac{\partial \boldsymbol{\omega}}{\partial \dot{\boldsymbol{\beta}}} \end{bmatrix} \end{bmatrix} \quad (\text{B.2})$$

The $\begin{bmatrix} \mathbf{F}_{sm} \\ \mathbf{T}_{sm} \end{bmatrix}$ term yields the strut acceleration effects.

$$\begin{bmatrix} \mathbf{F}_{sm} \\ \mathbf{T}_{sm} \end{bmatrix} = [\mathbf{M}_{strut}] \begin{Bmatrix} \mathbf{a}_c \\ \ddot{\mathbf{p}} \end{Bmatrix} \quad (\text{B.3})$$

$$\text{where } [\mathbf{M}_{strut}] = \begin{bmatrix} [\mathbf{M}_{ac}] & [\mathbf{M}_{aac}] \\ [\mathbf{M}_{aca}] & [\mathbf{M}_{a\alpha}] \end{bmatrix} \quad (\text{B.4})$$

$$[\mathbf{M}_{ac}] = -\sum_{i=1}^6 \left\{ \begin{aligned} & \left[\frac{\partial \mathbf{v}_E}{\partial \mathbf{v}_c} \right]^T (-m_E) \left[\frac{\partial \mathbf{v}_E}{\partial \mathbf{v}_c} \right] + \left[\frac{\partial \mathbf{v}_B}{\partial \mathbf{v}_c} \right]^T (-m_B) \left[\frac{\partial \mathbf{v}_B}{\partial \mathbf{v}_c} \right] \\ & + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \mathbf{v}_c} \right]^T (-1) [\mathbf{I}_E] \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \mathbf{v}_c} \right] + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \mathbf{v}_c} \right]^T (-1) [\mathbf{I}_B] \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \mathbf{v}_c} \right] \end{aligned} \right\}_i \quad (\text{B.5})$$

$$[\mathbf{M}_{ab}] = -\sum_{i=1}^6 \left\{ \begin{aligned} & \left[\frac{\partial \mathbf{v}_E}{\partial \mathbf{v}_c} \right]^T (-m_E) \left[\frac{\partial \mathbf{v}_E}{\partial \boldsymbol{\beta}} \right] + \left[\frac{\partial \mathbf{v}_B}{\partial \mathbf{v}_c} \right]^T (-m_B) \left[\frac{\partial \mathbf{v}_B}{\partial \boldsymbol{\beta}} \right] \\ & + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \mathbf{v}_c} \right]^T (-1) [\mathbf{I}_E] \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \boldsymbol{\beta}} \right] + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \mathbf{v}_c} \right]^T (-1) [\mathbf{I}_B] \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \boldsymbol{\beta}} \right] \end{aligned} \right\}_i \quad (\text{B.6})$$

$$[\mathbf{M}_{aa}] = -\sum_{i=1}^6 \left\{ \begin{aligned} & \left[\frac{\partial \mathbf{v}_E}{\partial \boldsymbol{\beta}} \right]^T (-m_E) \left[\frac{\partial \mathbf{v}_E}{\partial \mathbf{v}_c} \right] + \left[\frac{\partial \mathbf{v}_B}{\partial \boldsymbol{\beta}} \right]^T (-m_B) \left[\frac{\partial \mathbf{v}_B}{\partial \mathbf{v}_c} \right] \\ & + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \boldsymbol{\beta}} \right]^T (-1) [\mathbf{I}_E] \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \mathbf{v}_c} \right] + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \boldsymbol{\beta}} \right]^T (-1) [\mathbf{I}_B] \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \mathbf{v}_c} \right] \end{aligned} \right\}_i \quad (\text{B.7})$$

$$[\mathbf{M}_a] = -\sum_{i=1}^6 \left\{ \begin{aligned} & \left[\frac{\partial \mathbf{v}_E}{\partial \boldsymbol{\beta}} \right]^T (-m_E) \left[\frac{\partial \mathbf{v}_E}{\partial \boldsymbol{\beta}} \right] + \left[\frac{\partial \mathbf{v}_B}{\partial \boldsymbol{\beta}} \right]^T (-m_B) \left[\frac{\partial \mathbf{v}_B}{\partial \boldsymbol{\beta}} \right] \\ & + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \boldsymbol{\beta}} \right]^T (-1) [\mathbf{I}_E] \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \boldsymbol{\beta}} \right] + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \boldsymbol{\beta}} \right]^T (-1) [\mathbf{I}_B] \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \boldsymbol{\beta}} \right] \end{aligned} \right\}_i \quad (\text{B.8})$$

The Coriolis terms are grouped in $\begin{bmatrix} \mathbf{F}_{cor} \\ \mathbf{T}_{cor} \end{bmatrix}$.

where

$$\mathbf{F}_{cor} = \sum_{i=1}^6 \left\{ \begin{aligned} & \left[\frac{\partial \mathbf{v}_E}{\partial \mathbf{v}_c} \right]^T (-m_E) \left[2\dot{L}_E (\boldsymbol{\omega}_E \times \hat{\mathbf{s}}_3) - \left(\frac{2L_E \dot{L}}{L} \right) \left[[\hat{\mathbf{s}}_3 \times (\mathbf{v}_c + (\boldsymbol{\omega}_c \times \mathbf{r}_{pc}))] \times \hat{\mathbf{s}}_3 \right] \right] \\ & + \left[\frac{\partial \mathbf{v}_B}{\partial \mathbf{v}_c} \right]^T (-m_B) \left[-\left(\frac{2L_B \dot{L}}{L} \right) \left[[\hat{\mathbf{s}}_3 \times (\mathbf{v}_c + (\boldsymbol{\omega}_c \times \mathbf{r}_{pc}))] \times \hat{\mathbf{s}}_3 \right] \right] \\ & + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \mathbf{v}_c} \right]^T \left(\frac{2\dot{L}}{L} \right) [\mathbf{I}_E] \boldsymbol{\omega}_E + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \mathbf{v}_c} \right]^T \left(\frac{2\dot{L}}{L} \right) [\mathbf{I}_B] \boldsymbol{\omega}_B \end{aligned} \right\}_i \quad (\text{B.9})$$

$$\mathbf{T}_{cor} = \sum_{i=1}^6 \left\{ \begin{aligned} & \left[\frac{\partial \mathbf{v}_E}{\partial \dot{\boldsymbol{\beta}}} \right]^T (-m_E) \left[2\dot{L}_E (\boldsymbol{\omega}_E \times \hat{\mathbf{s}}_3) - \left(\frac{2L_E \dot{L}}{L} \right) \left[[\hat{\mathbf{s}}_3 \times (\mathbf{v}_c + (\boldsymbol{\omega}_c \times^w \mathbf{r}_{pc}))] \times \hat{\mathbf{s}}_3 \right] \right] \\ & + \left[\frac{\partial \mathbf{v}_B}{\partial \dot{\boldsymbol{\beta}}} \right]^T (-m_B) \left[- \left(\frac{2L_B \dot{L}}{L} \right) \left[[\hat{\mathbf{s}}_3 \times (\mathbf{v}_c + (\boldsymbol{\omega}_c \times^w \mathbf{r}_{pc}))] \times \hat{\mathbf{s}}_3 \right] \right] \\ & + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \dot{\boldsymbol{\beta}}} \right]^T \left(\frac{2\dot{L}}{L} \right) [\mathbf{I}_E] \boldsymbol{\omega}_E + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \dot{\boldsymbol{\beta}}} \right]^T \left(\frac{2\dot{L}}{L} \right) [\mathbf{I}_B] \boldsymbol{\omega}_B \end{aligned} \right\}_i \quad (\text{B.10})$$

The centripetal acceleration-like terms are grouped in $\begin{bmatrix} \mathbf{F}_{cen} \\ \mathbf{T}_{cen} \end{bmatrix}$.

where

$$\mathbf{F}_{cen} = \sum_{i=1}^6 \left\{ \begin{aligned} & \left[\frac{\partial \mathbf{v}_E}{\partial \mathbf{v}_c} \right]^T (-m_E) [\boldsymbol{\omega}_E \times (\boldsymbol{\omega}_E \times (L_E \hat{\mathbf{s}}_3)) + (\boldsymbol{\omega}_E^T \boldsymbol{\omega}_E) L \hat{\mathbf{s}}_3] \\ & + \left[\frac{\partial \mathbf{v}_B}{\partial \mathbf{v}_c} \right]^T (-m_B) [\boldsymbol{\omega}_B \times (\boldsymbol{\omega}_B \times (L_B \hat{\mathbf{s}}_3))] + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \mathbf{v}_c} \right]^T (-1) [\boldsymbol{\omega}_E \times [\mathbf{I}_E] \boldsymbol{\omega}_E] \\ & + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \mathbf{v}_c} \right]^T (-1) [\boldsymbol{\omega}_B \times [\mathbf{I}_B] \boldsymbol{\omega}_B] \\ & + \left[\frac{\partial \mathbf{v}_E}{\partial \mathbf{v}_c} \right]^T (-m_E) \left[\hat{\mathbf{s}}_3 \cdot \hat{\mathbf{s}}_3^T [\boldsymbol{\omega}_c \times (\boldsymbol{\omega}_c \times^w \mathbf{r}_{pc})] + \left(\frac{L_E}{L} \right) \left[[\hat{\mathbf{s}}_3 \times (\boldsymbol{\omega}_c \times (\boldsymbol{\omega}_c \times^w \mathbf{r}_{pc}))] \times \hat{\mathbf{s}}_3 \right] \right] \\ & + \left[\frac{\partial \mathbf{v}_B}{\partial \mathbf{v}_c} \right]^T (-m_B) \left[\left(\frac{L_B}{L} \right) \left[[\hat{\mathbf{s}}_3 \times (\boldsymbol{\omega}_c \times (\boldsymbol{\omega}_c \times^w \mathbf{r}_{pc}))] \times \hat{\mathbf{s}}_3 \right] \right] \\ & + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \mathbf{v}_c} \right]^T \left(-\frac{1}{L} \right) [\mathbf{I}_E] [\hat{\mathbf{s}}_3 \times (\boldsymbol{\omega}_c \times (\boldsymbol{\omega}_c \times^w \mathbf{r}_{pc}))] \\ & + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \mathbf{v}_c} \right]^T \left(-\frac{1}{L} \right) [\mathbf{I}_B] [\hat{\mathbf{s}}_3 \times (\boldsymbol{\omega}_c \times (\boldsymbol{\omega}_c \times^w \mathbf{r}_{pc}))] \end{aligned} \right\}_i \quad (\text{B.11})$$

$$\mathbf{T}_{con} = \sum_{i=1}^6 \left\{ \begin{aligned} & \left[\frac{\partial \mathbf{v}_E}{\partial \dot{\boldsymbol{\beta}}} \right]^T (-m_E) [\boldsymbol{\omega}_E \times (\boldsymbol{\omega}_E \times (L_E \hat{\mathbf{s}}_3)) + (\boldsymbol{\omega}_E^T \boldsymbol{\omega}_E) L_E \hat{\mathbf{s}}_3] + \left[\frac{\partial \mathbf{v}_B}{\partial \dot{\boldsymbol{\beta}}} \right]^T (-m_B) [\boldsymbol{\omega}_B \times (\boldsymbol{\omega}_B \times (L_B \hat{\mathbf{s}}_3))] \\ & + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \dot{\boldsymbol{\beta}}} \right]^T (-1) [\boldsymbol{\omega}_E \times [\mathbf{I}_E] \boldsymbol{\omega}_E] + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \dot{\boldsymbol{\beta}}} \right]^T (-1) [\boldsymbol{\omega}_B \times [\mathbf{I}_B] \boldsymbol{\omega}_B] \\ & + \left[\frac{\partial \mathbf{v}_E}{\partial \dot{\boldsymbol{\beta}}} \right]^T (-m_E) \left[\hat{\mathbf{s}}_3 \cdot \hat{\mathbf{s}}_3^T [\boldsymbol{\omega}_c \times (\boldsymbol{\omega}_c \times \mathbf{r}_{pc})] + \left(\frac{L_E}{L} \right) [[\hat{\mathbf{s}}_3 \times (\boldsymbol{\omega}_c \times (\boldsymbol{\omega}_c \times \mathbf{r}_{pc}))] \times \hat{\mathbf{s}}_3] \right] \\ & + \left[\frac{\partial \mathbf{v}_B}{\partial \dot{\boldsymbol{\beta}}} \right]^T (-m_B) \left[\left(\frac{L_B}{L} \right) [[\hat{\mathbf{s}}_3 \times (\boldsymbol{\omega}_c \times (\boldsymbol{\omega}_c \times \mathbf{r}_{pc}))] \times \hat{\mathbf{s}}_3] \right] \\ & + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \dot{\boldsymbol{\beta}}} \right]^T \left(-\frac{1}{L} \right) [\mathbf{I}_E] [\hat{\mathbf{s}}_3 \times (\boldsymbol{\omega}_c \times (\boldsymbol{\omega}_c \times \mathbf{r}_{pc}))] \\ & + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \dot{\boldsymbol{\beta}}} \right]^T \left(-\frac{1}{L} \right) [\mathbf{I}_B] [\hat{\mathbf{s}}_3 \times (\boldsymbol{\omega}_c \times (\boldsymbol{\omega}_c \times \mathbf{r}_{pc}))] \\ & + \left[\frac{\partial \boldsymbol{\omega}_c}{\partial \dot{\boldsymbol{\beta}}} \right]^T (-1) [\boldsymbol{\omega}_c \times [\mathbf{I}_c] \boldsymbol{\omega}_c] \end{aligned} \right\} \quad (\text{B.12})$$

The gravitational dependent terms for the platform are grouped in $\begin{bmatrix} \mathbf{F}_{grp} \\ \mathbf{T}_{grp} \end{bmatrix}$.

where

$$\mathbf{F}_{grp} = \left[\frac{\partial \mathbf{v}_c}{\partial \dot{\mathbf{v}}_c} \right]^T m_p \mathbf{g} \quad (\text{B.13})$$

$$\mathbf{T}_{grp} = 0 \quad (\text{B.14})$$

The gravitation dependent terms in equation (2.55) are grouped in $\begin{bmatrix} \mathbf{F}_{grs} \\ \mathbf{T}_{grs} \end{bmatrix}$

where

$$\mathbf{F}_{grs} = \sum_{i=1}^6 \left\{ \left[\frac{\partial \mathbf{v}_E}{\partial \dot{\mathbf{v}}_c} \right]^T m_E \mathbf{g} + \left[\frac{\partial \mathbf{v}_B}{\partial \dot{\mathbf{v}}_c} \right]^T m_B \mathbf{g} \right\}_i \quad (\text{B.15})$$

$$\mathbf{T}_{grs} = \sum_{i=1}^6 \left\{ \left[\frac{\partial \mathbf{v}_E}{\partial \boldsymbol{\beta}} \right]^T m_E \mathbf{g} + \left[\frac{\partial \mathbf{v}_B}{\partial \boldsymbol{\beta}} \right]^T m_B \mathbf{g} \right\}_i \quad (\text{B.16})$$

One of the sources of nonlinearities are joint frictions which are denoted by the $\begin{bmatrix} \mathbf{F}_{frict} \\ \mathbf{T}_{frict} \end{bmatrix}$

and are dependent on the PKM instantaneous configuration and velocity.

where

$$\mathbf{F}_{frict} = (-1) \left[(\hat{\mathbf{s}}_1)_1 \quad \dots \quad (\hat{\mathbf{s}}_6)_6 \right] \begin{Bmatrix} F_{\zeta_1} \\ \vdots \\ F_{\zeta_6} \end{Bmatrix} + \sum_{i=1}^6 \left\{ \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \mathbf{v}_c} \right]^T \boldsymbol{\tau}_{\zeta_r} + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \mathbf{v}_c} \right]^T \boldsymbol{\tau}_{\zeta_s} \right\}_i \quad (\text{B.17})$$

$$\mathbf{T}_{frict} = \sum_{i=1}^6 \left\{ [\mathbf{U}] \left({}^* \mathbf{r}_{pc} \times \hat{\mathbf{s}}_3 \right) (-\mathbf{F}_{\zeta}) + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \boldsymbol{\beta}} \right]^T \boldsymbol{\tau}_{\zeta_r} + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \boldsymbol{\beta}} \right]^T \boldsymbol{\tau}_{\zeta_s} + \left[\frac{\partial \boldsymbol{\omega}_c}{\partial \boldsymbol{\beta}} \right]^T [-\boldsymbol{\tau}_{\zeta_s}] \right\}_i \quad (\text{B.18})$$

The friction terms $\mathbf{F}_{\zeta_i}$ are modeled as modified friction.

The $\begin{bmatrix} \mathbf{F}_{ext} \\ \mathbf{T}_{ext} \end{bmatrix}$ consists of the external loads.

where

$$\mathbf{F}_{ext} = \sum_{i=1}^6 \left\{ \left[\frac{\partial \mathbf{v}_E}{\partial \mathbf{v}_c} \right]^T \mathbf{F}_{ext_E} + \left[\frac{\partial \mathbf{v}_B}{\partial \mathbf{v}_c} \right]^T \mathbf{F}_{ext_B} + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \mathbf{v}_c} \right]^T \boldsymbol{\tau}_{ext_E} + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \mathbf{v}_c} \right]^T \boldsymbol{\tau}_{ext_B} + \left[\frac{\partial \mathbf{v}_c}{\partial \mathbf{v}_c} \right]^T \mathbf{F}_{ext_c} \right\}_i \quad (\text{B.19})$$

$$\mathbf{T}_{ext} = \sum_{i=1}^6 \left\{ \left[\frac{\partial \mathbf{v}_E}{\partial \boldsymbol{\beta}} \right]^T \mathbf{F}_{ext_E} + \left[\frac{\partial \mathbf{v}_B}{\partial \boldsymbol{\beta}} \right]^T \mathbf{F}_{ext_B} + \left[\frac{\partial \boldsymbol{\omega}_E}{\partial \boldsymbol{\beta}} \right]^T \boldsymbol{\tau}_{ext_E} + \left[\frac{\partial \boldsymbol{\omega}_B}{\partial \boldsymbol{\beta}} \right]^T \boldsymbol{\tau}_{ext_B} \right\}_i + \left[\frac{\partial \boldsymbol{\omega}_c}{\partial \boldsymbol{\beta}} \right]^T \boldsymbol{\tau}_{ext_c} \quad (\text{B.20})$$

The left hand side of the equation

$$\mathbf{JF}_a = \left\{ \begin{bmatrix} \mathbf{JF}_a \\ \mathbf{JF}_a \end{bmatrix} \right\} \quad (\text{B.21})$$

where

$$[\mathbf{JF}_a]_{upper} = \begin{bmatrix} (\hat{\mathbf{s}}_3)_1 & \vdots & \dots & \vdots & (\hat{\mathbf{s}}_3)_6 \end{bmatrix} \begin{bmatrix} \mathbf{F}_{a_1} \\ \vdots \\ \mathbf{F}_{a_6} \end{bmatrix} \quad (\text{B.22})$$

$$[\mathbf{JF}_a]_{lower} = \begin{bmatrix} [\mathbf{U}]^T (\mathbf{r}_{pc} \times \hat{\mathbf{s}}_3)_1 & \vdots & \dots & \vdots & [\mathbf{U}]^T (\mathbf{r}_{pc} \times \hat{\mathbf{s}}_3)_6 \end{bmatrix} \begin{bmatrix} \mathbf{F}_{a_1} \\ \vdots \\ \mathbf{F}_{a_6} \end{bmatrix} \quad (\text{B.23})$$

$$\text{and } \mathbf{F}_a = \begin{bmatrix} \mathbf{F}_{a_1} \\ \vdots \\ \mathbf{F}_{a_6} \end{bmatrix}, \mathbf{J} = \begin{bmatrix} (\hat{\mathbf{s}}_3)_1 & \vdots & \dots & \vdots & (\hat{\mathbf{s}}_3)_6 \\ [\mathbf{U}]^T (\mathbf{r}_{pc} \times \hat{\mathbf{s}}_3)_1 & \vdots & \dots & \vdots & [\mathbf{U}]^T (\mathbf{r}_{pc} \times \hat{\mathbf{s}}_3)_6 \end{bmatrix} \quad (\text{B.24})$$

$$\text{Additionally, } \begin{bmatrix} \mathbf{F}_{pm} \\ \mathbf{T}_{pm} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_{sm} \\ \mathbf{T}_{sm} \end{bmatrix} = [\mathbf{M}] \begin{bmatrix} \mathbf{a}_c \\ \dot{\mathbf{p}} \end{bmatrix} = \left[[\mathbf{M}_p] + [\mathbf{M}_{strut}] \right] \begin{bmatrix} \mathbf{a}_c \\ \dot{\mathbf{p}} \end{bmatrix} \quad (\text{B.25})$$

From section 2.4, equation (2.71),

$$\begin{aligned} T_k &= \frac{1}{2} \begin{bmatrix} \mathbf{v}_c^T, \boldsymbol{\omega}_c^T \end{bmatrix} \begin{bmatrix} \tilde{M}_{11} & \tilde{M}_{12} \\ \tilde{M}_{21} & \tilde{M}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{v}_c \\ \boldsymbol{\omega}_c \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} \mathbf{v}_c^T, ([\mathbf{U}]\dot{\mathbf{p}})^T \end{bmatrix} \begin{bmatrix} \tilde{M}_{11} & \tilde{M}_{12} \\ \tilde{M}_{21} & \tilde{M}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{v}_c \\ [\mathbf{U}]\dot{\mathbf{p}} \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} \mathbf{v}_c^T, (\dot{\mathbf{p}})^T \end{bmatrix} \begin{bmatrix} [\mathbf{I}] & [0] \\ [0] & [\mathbf{U}]^T \end{bmatrix} \begin{bmatrix} \tilde{M}_{11} & \tilde{M}_{12} \\ \tilde{M}_{21} & \tilde{M}_{22} \end{bmatrix} \begin{bmatrix} [\mathbf{I}] & [0] \\ [0] & [\mathbf{U}] \end{bmatrix} \begin{bmatrix} \mathbf{v}_c \\ \dot{\mathbf{p}} \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} \mathbf{v}_c^T, (\dot{\mathbf{p}})^T \end{bmatrix} \begin{bmatrix} \tilde{M}_{11} & \tilde{M}_{12} \\ [\mathbf{U}]^T \tilde{M}_{21} & [\mathbf{U}]^T \tilde{M}_{22} \end{bmatrix} \begin{bmatrix} [\mathbf{I}] & [0] \\ [0] & [\mathbf{U}] \end{bmatrix} \begin{bmatrix} \mathbf{v}_c \\ \dot{\mathbf{p}} \end{bmatrix} \end{aligned} \quad (\text{B.26})$$

$$= \frac{1}{2} [\mathbf{v}_c^T, (\dot{\hat{\mathbf{p}}})^T] \begin{bmatrix} \tilde{M}_{11} & \tilde{M}_{12}[\mathbf{U}] \\ [\mathbf{U}]^T \tilde{M}_{21} & [\mathbf{U}]^T \tilde{M}_{22}[\mathbf{U}] \end{bmatrix} \begin{bmatrix} \mathbf{v}_c \\ \dot{\hat{\mathbf{p}}} \end{bmatrix}$$

$$\text{and } T_k = \frac{1}{2} [\mathbf{v}_c^T, (\dot{\hat{\mathbf{p}}})^T] [\mathbf{M}] \begin{bmatrix} \mathbf{v}_c \\ \dot{\hat{\mathbf{p}}} \end{bmatrix} \quad (\text{B.27})$$

$$\text{where } [\mathbf{M}] = [\mathbf{M}_p] + [\mathbf{M}_{\text{strut}}]$$

Then

$$\begin{aligned} T_k &= \frac{1}{2} [\mathbf{v}_c^T, (\dot{\hat{\mathbf{p}}})^T] \begin{bmatrix} m_p [\mathbf{I}] & [0] \\ [0] & \left[\frac{\partial \boldsymbol{\omega}}{\partial \boldsymbol{\beta}} \right]^T [\mathbf{I}_p] \left[\frac{\partial \boldsymbol{\omega}}{\partial \boldsymbol{\beta}} \right] \end{bmatrix} + \begin{bmatrix} [\mathbf{M}_{ac}] & [\mathbf{M}_{aac}] \\ [\mathbf{M}_{aca}] & [\mathbf{M}_a] \end{bmatrix} \begin{bmatrix} \mathbf{v}_c \\ \dot{\hat{\mathbf{p}}} \end{bmatrix} \\ &= \frac{1}{2} [\mathbf{v}_c^T, (\dot{\hat{\mathbf{p}}})^T] \begin{bmatrix} m_p [\mathbf{I}] + [\mathbf{M}_{ac}] & [\mathbf{M}_{aac}] \\ [\mathbf{M}_{aca}] & \left[\frac{\partial \boldsymbol{\omega}}{\partial \boldsymbol{\beta}} \right]^T [\mathbf{I}_p] \left[\frac{\partial \boldsymbol{\omega}}{\partial \boldsymbol{\beta}} \right] + [\mathbf{M}_a] \end{bmatrix} \begin{bmatrix} \mathbf{v}_c \\ \dot{\hat{\mathbf{p}}} \end{bmatrix} \end{aligned} \quad (\text{B.28})$$

Thus,

$$\tilde{M}_{11} = m_p [\mathbf{I}] + [\mathbf{M}_{ac}] \quad (\text{B.29})$$

$$\tilde{M}_{12} = [\mathbf{M}_{aac}] [\mathbf{U}]^{-1} \quad (\text{B.30})$$

$$\tilde{M}_{21} = [\mathbf{U}]^T [\mathbf{M}_{aca}] \quad (\text{B.31})$$

$$\tilde{M}_{22} = [\mathbf{U}]^T \left\{ \left[\frac{\partial \boldsymbol{\omega}}{\partial \boldsymbol{\beta}} \right]^T [\mathbf{I}_p] \left[\frac{\partial \boldsymbol{\omega}}{\partial \boldsymbol{\beta}} \right] + [\mathbf{M}_a] \right\} [\mathbf{U}]^{-1} \quad (\text{B.32})$$

The derivation of the Pseudo-Jacobian matrix: In equation (2.40), the F_a terms are shown on $\mathbf{F}_e$ (equation (2.42)) and $\mathbf{F}_B$ (equation (2.43)). Thus, the Pseudo-Jacobian matrix is calculated from the following equations.

$$\begin{aligned} \frac{\partial \mathbf{v}_e}{\partial \mathbf{v}_c} \cdot F_a \hat{\mathbf{s}}_3 &= \left((\hat{\mathbf{s}}_3 \hat{\mathbf{s}}_3^T) + \left(\frac{L_e}{L} \right) \left[(\hat{\mathbf{s}}_3 \times \hat{i}) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times \hat{j}) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times \hat{k}) \times \hat{\mathbf{s}}_3 \right] \right) \cdot F_a \hat{\mathbf{s}}_3 \\ &= (\hat{\mathbf{s}}_3 \hat{\mathbf{s}}_3^T) \cdot F_a \hat{\mathbf{s}}_3 \\ &= F_a \hat{\mathbf{s}}_3 \end{aligned} \quad (\text{B.33})$$

$$\frac{\partial \mathbf{v}_B}{\partial \mathbf{v}_c} \cdot F_a \hat{\mathbf{s}}_3 = \left(\left(\frac{L_B}{L} \right) \left[(\hat{\mathbf{s}}_3 \times \hat{i}) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times \hat{j}) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times \hat{k}) \times \hat{\mathbf{s}}_3 \right] \right) \cdot F_a \hat{\mathbf{s}}_3 = [\mathbf{0}] \quad (\text{B.34})$$

$$\begin{aligned} \frac{\partial \mathbf{v}_e}{\partial \mathbf{p}} \cdot F_a \hat{\mathbf{s}}_3 &= \hat{\mathbf{s}}_3 \left[({}^u \mathbf{r}_{pc} \times \hat{\mathbf{s}}_3)^T [\mathbf{U}] \right] \\ &+ \left(\frac{L_e}{L} \right) \left[(\hat{\mathbf{s}}_3 \times (\left[\mathbf{U} \right]_{Col1} \times {}^u \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times (\left[\mathbf{U} \right]_{Col2} \times {}^u \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times (\left[\mathbf{U} \right]_{Col3} \times {}^u \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \right] \cdot F_a \hat{\mathbf{s}}_3 \\ &= \hat{\mathbf{s}}_3 \left[({}^u \mathbf{r}_{pc} \times \hat{\mathbf{s}}_3)^T [\mathbf{U}] \right] \cdot F_a \hat{\mathbf{s}}_3 \\ &= \left[({}^u \mathbf{r}_{pc} \times \hat{\mathbf{s}}_3)^T [\mathbf{U}] \right] \hat{\mathbf{s}}_3^T \cdot F_a \hat{\mathbf{s}}_3 \\ &= [\mathbf{U}]^T ({}^u \mathbf{r}_{pc} \times \hat{\mathbf{s}}_3) F_a \\ &\therefore \left(\frac{L_e}{L} \right) \left[(\hat{\mathbf{s}}_3 \times (\left[\mathbf{U} \right]_{Col1} \times {}^u \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times (\left[\mathbf{U} \right]_{Col2} \times {}^u \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times (\left[\mathbf{U} \right]_{Col3} \times {}^u \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \right] \cdot F_a \hat{\mathbf{s}}_3 = [\mathbf{0}] \end{aligned} \quad (\text{B.35})$$

$$\frac{\partial \mathbf{v}_B}{\partial \mathbf{p}} \cdot F_a \hat{\mathbf{s}}_3 = \left(\frac{L_B}{L} \right) \left[(\hat{\mathbf{s}}_3 \times (\left[\mathbf{U} \right]_{Col1} \times {}^u \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times (\left[\mathbf{U} \right]_{Col2} \times {}^u \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \quad : \quad (\hat{\mathbf{s}}_3 \times (\left[\mathbf{U} \right]_{Col3} \times {}^u \mathbf{r}_{pc})) \times \hat{\mathbf{s}}_3 \right] \cdot F_a \hat{\mathbf{s}}_3 = [\mathbf{0}] \quad (\text{B.36})$$

Therefore, the Pseudo-Jacobian matrix is forming with (B.33) and (B.35) as shown on equation (B.21) – (B.24)

APPENDIX C FIGURES FOR THE COMPARISON BETWEEN FORCE-BASED MODEL AND ENERGY-BASED MODEL WITH USING THE NOISY DATA

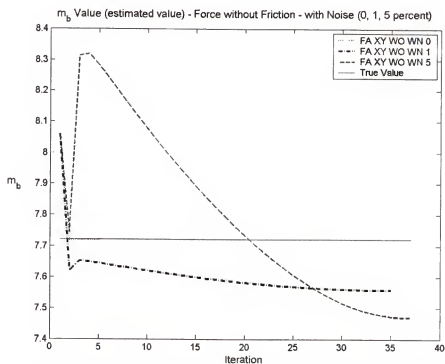
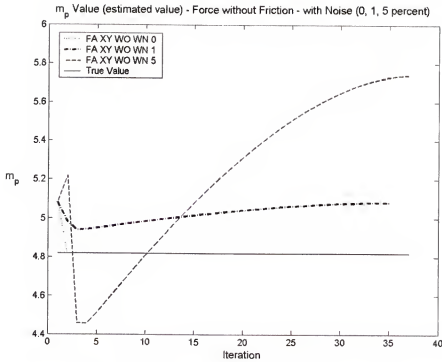
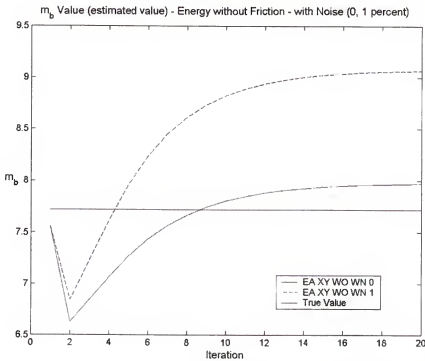
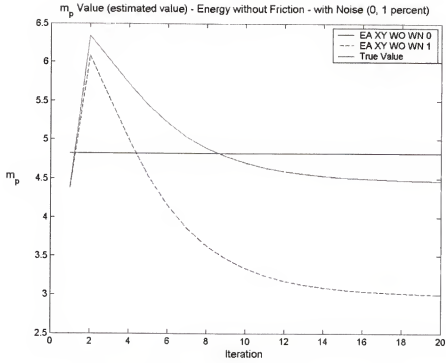
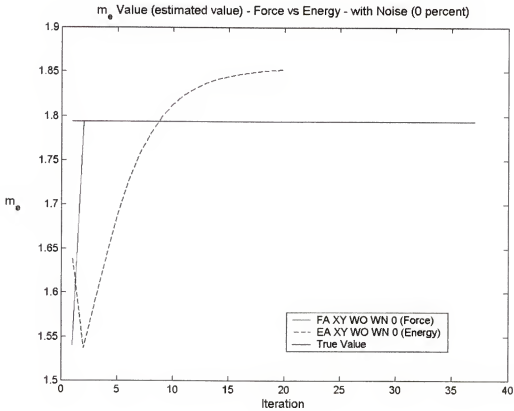
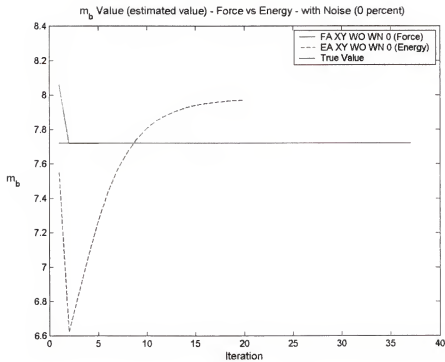
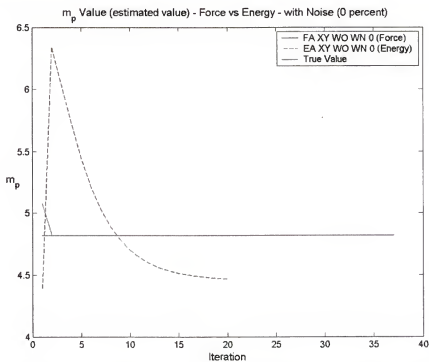
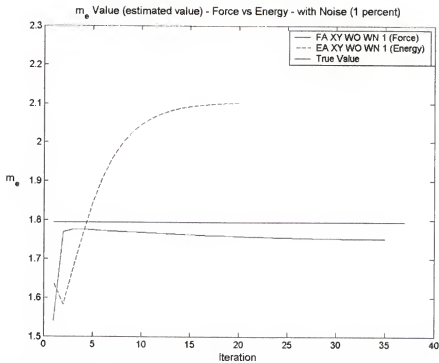
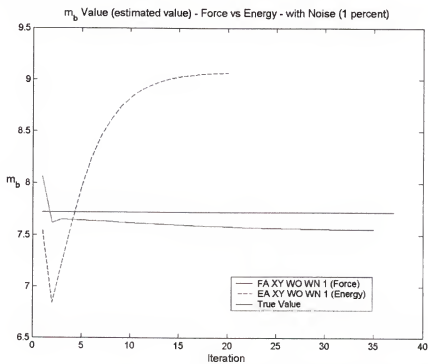


Figure C.1 m_b Value (0, 1 and 5 % Noise, Force-Based Model)

Figure C.2 m_p Value (0, 1 and 5 % Noise, Force-Based Model)Figure C.3 m_b Value (0 and 1 % Noise, Energy-Based Model)

Figure C.4 m_p Value (0 and 1 % Noise, Energy-Based Model)Figure C.5 m_e Value (0 % Noise, Force-Based vs. Energy-Based Model)

Figure C.6 m_b Value (0 % Noise, Force-Based vs. Energy-Based Modeling)Figure C.7 m_p Value (0 % Noise, Force- Based vs. Energy-Based Modeling)

Figure C.8 m_e Value (1 % Noise, Force- Based vs. Energy-Based Modeling)Figure C.9 m_b Value (1 % Noise, Force- Based vs. Energy-Based Modeling)

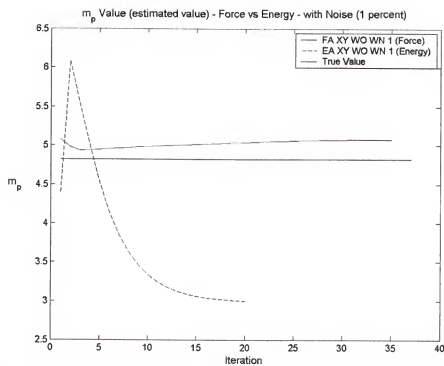


Figure C.10 m_p Value (1 % Noise, Force- Based vs. Energy-Based Modeling)

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BIOGRAPHICAL SKETCH

Young Hoon Oh was born November 21, 1971, in Seoul, South Korea. His early days were spent under the love, guidance and nurturing of his loving parents. He received a Bachelor of Science degree in mechanical engineering in February 1997. He enrolled in graduate school at the University of Florida in August 1997. His master's degree in mechanical engineering was received in August 2000. Young Hoon began his Ph.D program also in August 2000.

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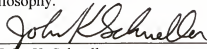
Carl D. Crane, III
Professor of Mechanical and Aerospace
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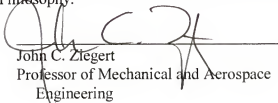
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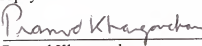
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